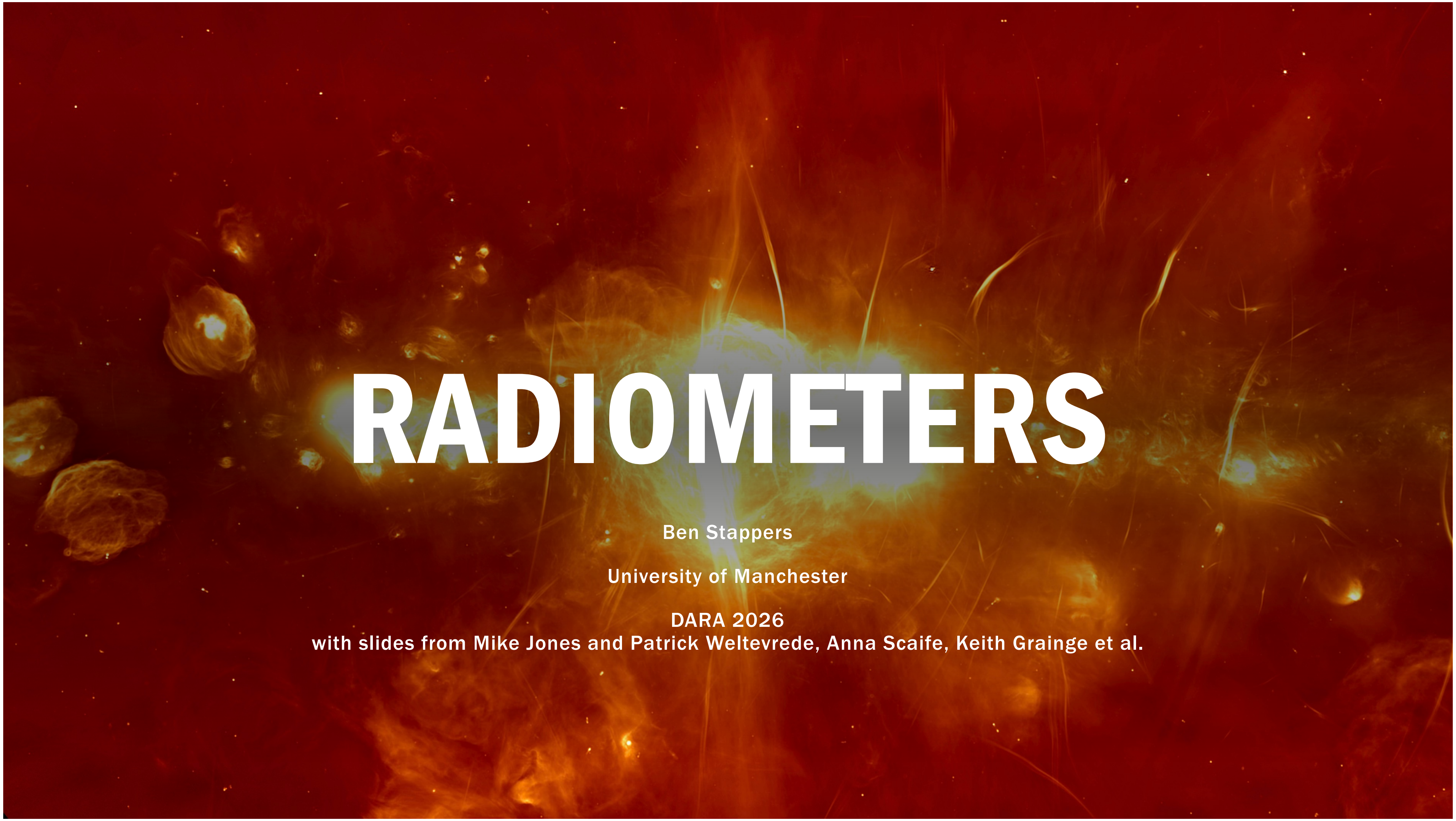

RADIOMETERS

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DARA 2026
with slides from Mike Jones and Patrick Weltevrede, Anna Scaife, Keith Grainge et al.

Electromagnetic Radiation

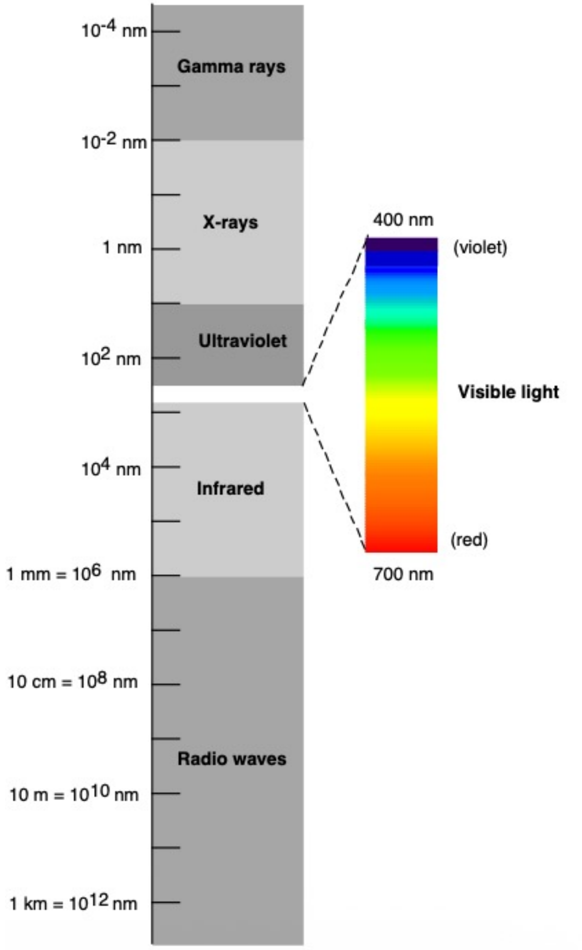
- Visible light has wavelengths 400 –700 nm
- Radio has wavelengths longer than 1 mm (not a strict definition)
- Sub-mm (0.1 – 1 mm) can also use radio techniques
- Millimetre (1 – 10 mm)
- Microwave (1 – 10s cm)

Frequency = speed of light / wavelength

– 10 GHz == 3 cm

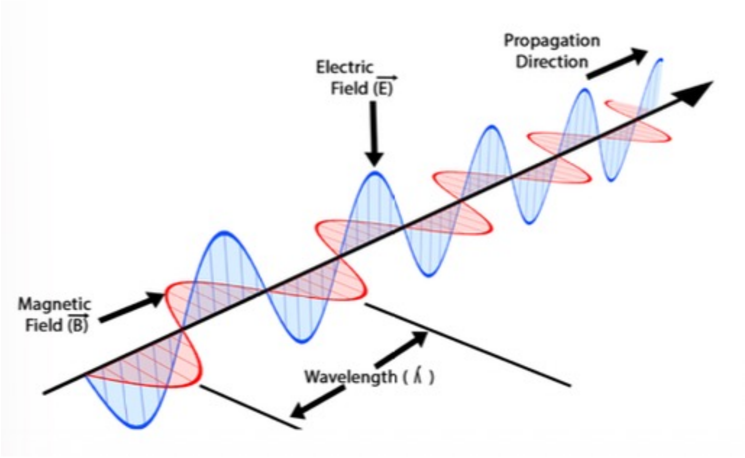
– 1 GHz == 30 cm

– 100 MHz = 300 cm

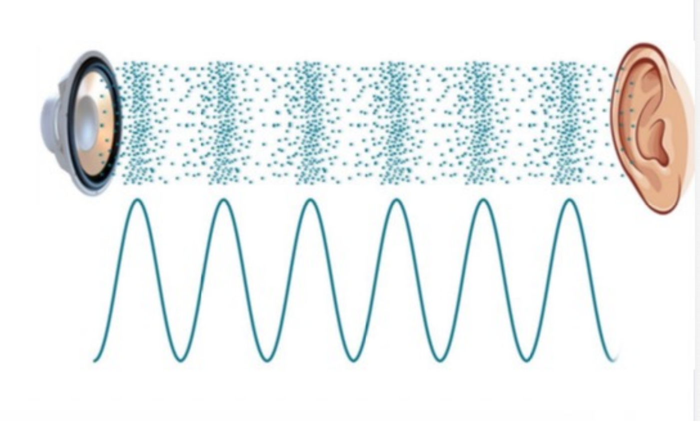


Radio versus Sound

Radio travels in a vacuum
Radio waves are transverse
Radio waves have polarization (direction of transverse field)

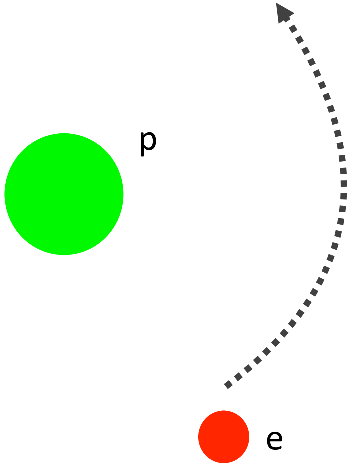


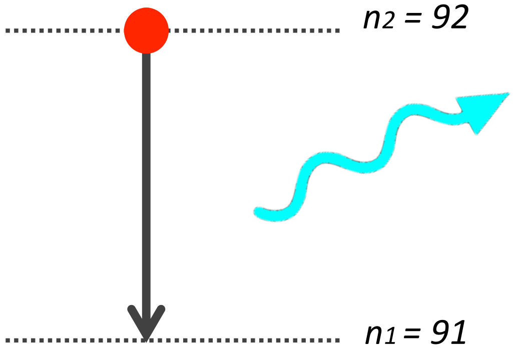
Sound is waves in a medium (e.g. air, water, solids)
Sound waves are longitudinal
Sound waves are not polarised

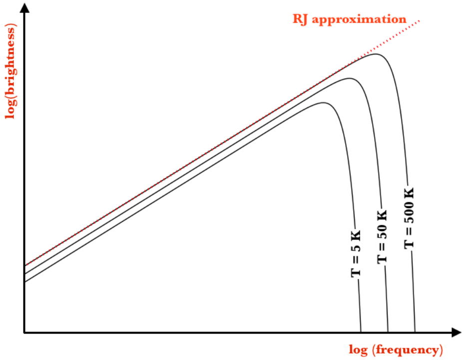


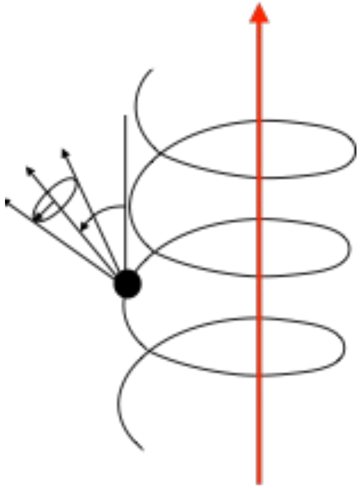
Electromagnetic Radiation

- Radio, light, X-rays, infra-red etc… are all examples of ***electromagnetic radiation***
- They can be described by Maxwell's equations that link electric and magnetic fields
- A changing electric field creates a magnetic field and a changing magnetic field creates and electric field.
- As soon as you create a varying electric field, you generate electromagnetic radiation.



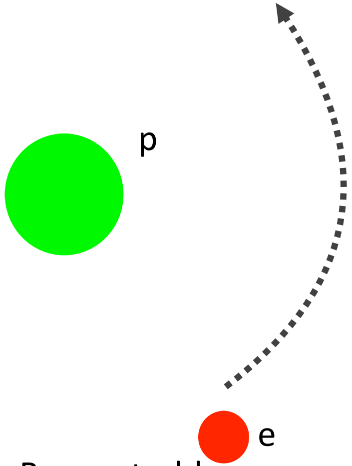




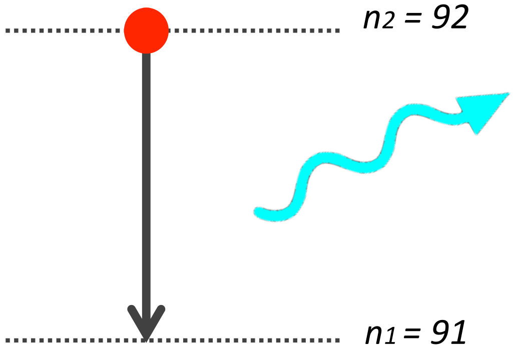


Electromagnetic Radiation

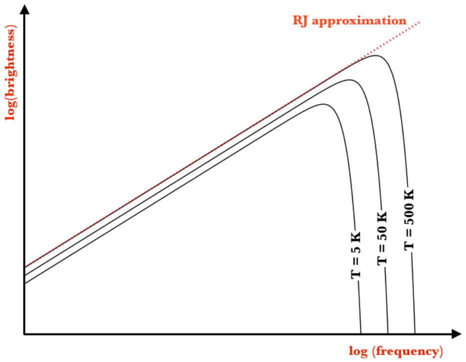
- Radio, light, X-rays, infra-red etc… are all examples of ***electromagnetic radiation***
- They can be described by Maxwell’s equations that link electric and magnetic fields
- A changing electric field creates a magnetic field and a changing magnetic field creates and electric field.
- As soon as you create a varying electric field, you generate electromagnetic radiation.
- Artificially – shake electrons in a wire using a voltage
- Receiving is exactly the same process in reverse!



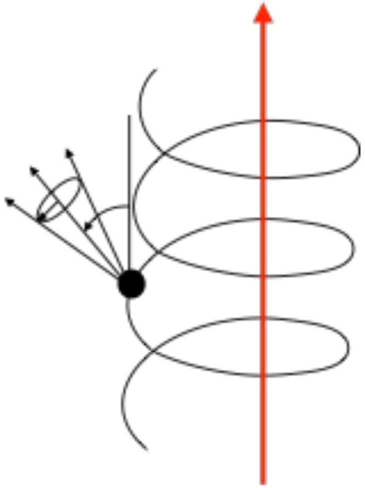
Bremsstrahlung



Spectral Line



Blackbody (Thermal)



Synchrotron

Luminosity

- An object's **luminosity**, L, is the total amount of power that it is emitting in EM radiation.

  The units of luminosity are therefore Watt

- Considering a spherical shell at distance R from the object, the total power in EM radiation flowing through this shell is also L
- The **specific luminosity** L_ν (or power per unit frequency P_ν) is the luminosity density with respect to frequency
  - The units of specific luminosity are W Hz^{-1}
  - The power emitted between $\nu \rightarrow \nu + \mathrm{d}\nu$ is $L_\nu \, \mathrm{d}\nu$
- Since the EM radiation has a spectral dependence, the total power over all frequencies (**bolometric luminosity**) is:

$$L = \int_0^\infty P_\nu \, \mathrm{d}\nu$$

Flux Density

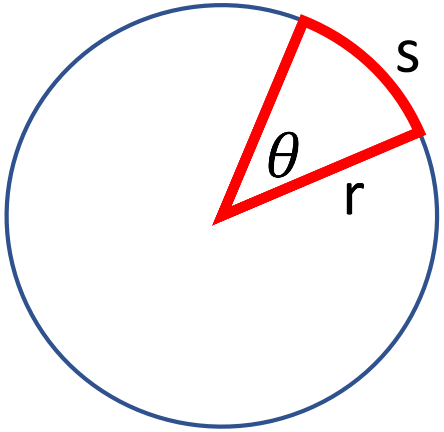
- The flux density F_ν is the power density of EM radiation passing through a unit area
  - The units of flux density are therefore W m^{-2} Hz^{-1}
- Consider an object emitting isotropically with specific luminosity L_ν, then at a distance R the flux density will be given by

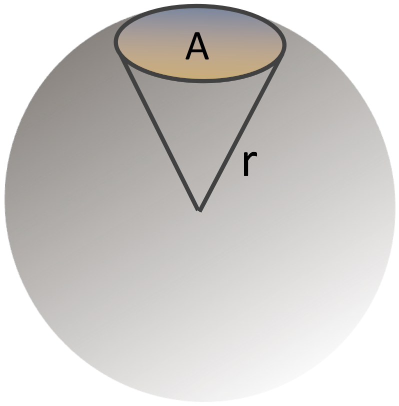
$$L_\nu = \int F_\nu \, \mathrm{d}a \quad \Rightarrow \quad F_\nu = \frac{L_\nu}{4\pi R^2}$$

- To find the total power density received by a dish antenna, multiply the flux density by the dish's area
- Note that scientific literature / text books / lecture notes / me are often rather slack and use the term **flux** rather than the correct **flux density**

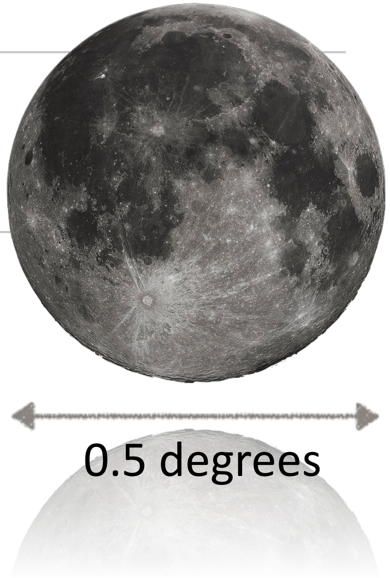
Aside: Solid Angles

- The definition of an angle in radians: *the arc of a circle divided by its radius:* $\theta = s/r$
- Solid angle is the 2-D analogue of angle: A solid angle Ω is the *area* A on the surface of a sphere divided by its radius r squared: $\Omega = A/r^2$
- Mathematically, the solid angle does not have units, but for practical reasons, we assign virtual units of “steradians” or sr.
- The surface area of a sphere is $4\pi r^2$ hence its solid angle is $\Omega = 4\pi$ sr.
- An isotropic emitter will therefore radiate into a solid angle of 4π sr





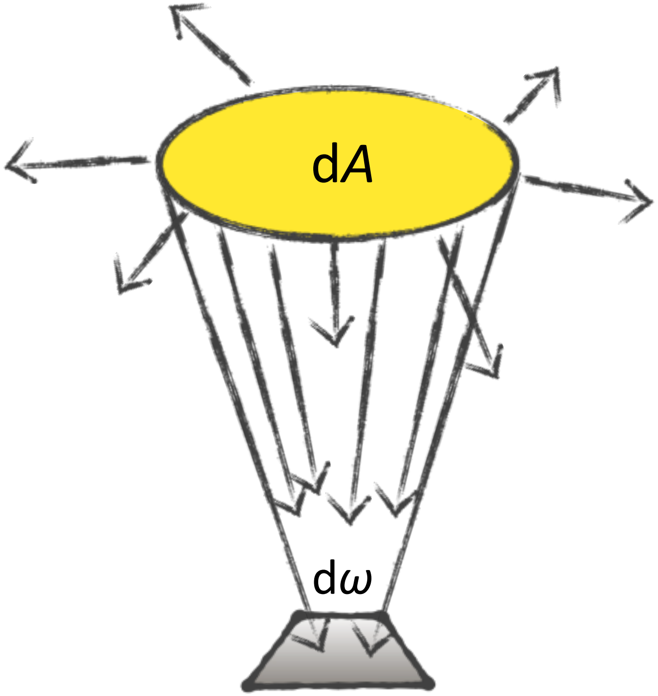
Solid Angles - Example



- The Moon has an angular diameter of *d* = 0.5 deg, so the solid angle it subtends is (small angle approx.) $\Omega = \pi\,(d/2)^2 = \pi\,(0.5/2)^2 = 0.196\ \mathrm{deg}^2$
- Alternatively, working in radians and sterad
  - The Moon's angular diameter is $\pi/180 \times 0.5 = 0.00873$ rad
  - The solid angle it subtends is $\Omega = \pi\,(0.00873/2)^2 = 6{\times}10^{-5}$ sr
- Another approach – converting from deg^2 to sr
  - $1\ \mathrm{sr} = (1\ \mathrm{rad})^2 = (180/\pi)^2\ \mathrm{deg}^2 = 3282.8\ \mathrm{deg}^2$
  - $\Omega = 0.196\ \mathrm{deg}^2 = 0.196/3282.8\ \mathrm{sr} = 6 \times 10^{-5}$ sr

Specific Intensity

Specific intensity I_ν: the power *leaving* unit area **d*A*** of an emitting region, at a frequency ν per unit frequency interval **dν**, into a solid angle **dω** heading towards the collector.



Specific intensity I_ν (often sloppily referred to as "intensity") is a fundamental property related to the physics of the emission process. It has units of **W m^{-2} Hz^{-1} sr^{-1}**

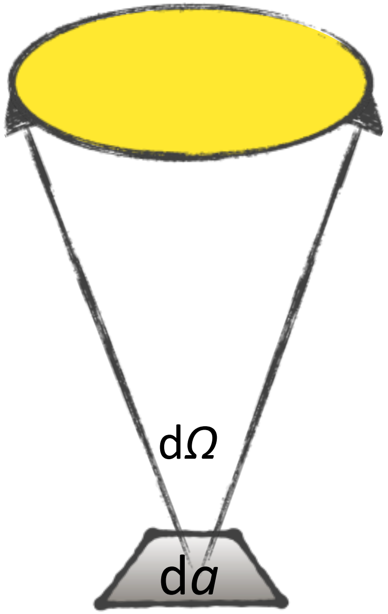
Picture: Radio photons leaving an extended emitting region. A few are heading towards the collector system and constitute an energy flow towards it – the vast majority miss the collector!

The power received over a frequency interval dν from area dA of the apparent surface of the source into a solid angle dω towards the collector system: $P_{em} = I_\nu \, dA \, d\nu \, d\omega$

Note: astronomical sources are effectively at infinity – so 3-D objects appear as 2-D surfaces and the energy stream can be described in terms of plane waves fronts/rays.

Surface Brightness

Surface Brightness B_ν: The power *arriving* per unit area **d*a*** of the collector system at a frequency ***ν*** per unit frequency interval **d*ν*** from a solid angle **d*Ω*** of the emitting region.



Same units as specific intensity: **W m^{-2} Hz^{-1} sr^{-1}**

Area d*a* of the collector receives power: $P_{rec} = B_\nu \, da \, d\nu \, d\Omega$

Surface brightness and flux

- Consider the flux density passing through a unit area due to EM radiation approaching from an angle θ with respect to the area's normal.

$$F_\nu = \int B_\nu(\theta,\phi)\cos\theta \mathrm{d}\Omega$$

- The total power received by an antenna is therefore

$$P_{\mathrm{rec}} = \int B_\nu(\theta,\phi)\cos\theta \mathrm{d}A\mathrm{d}\nu\mathrm{d}\Omega$$

- Since θ is often small (detector is aimed straight at source), the cos θ term can often be ignored

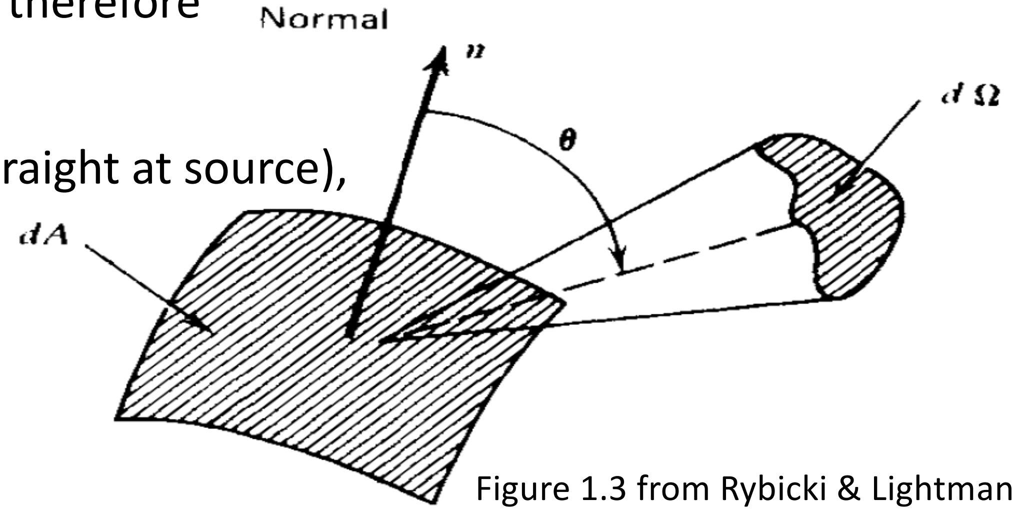


Figure 1.3 from Rybicki & Lightman

Surface brightness and flux – Alternative variables

$$S = \int I_\nu \, d\Omega$$

Flux Density

Brightness

- This is one of the most important definitions in radio astronomy!
- Flux density is the integral of brightness over solid angle.
- Brightness is flux density per solid angle.

Radio astronomers always talk about flux density and brightness (or brightness temperature…coming up).

It's easy to get confused!

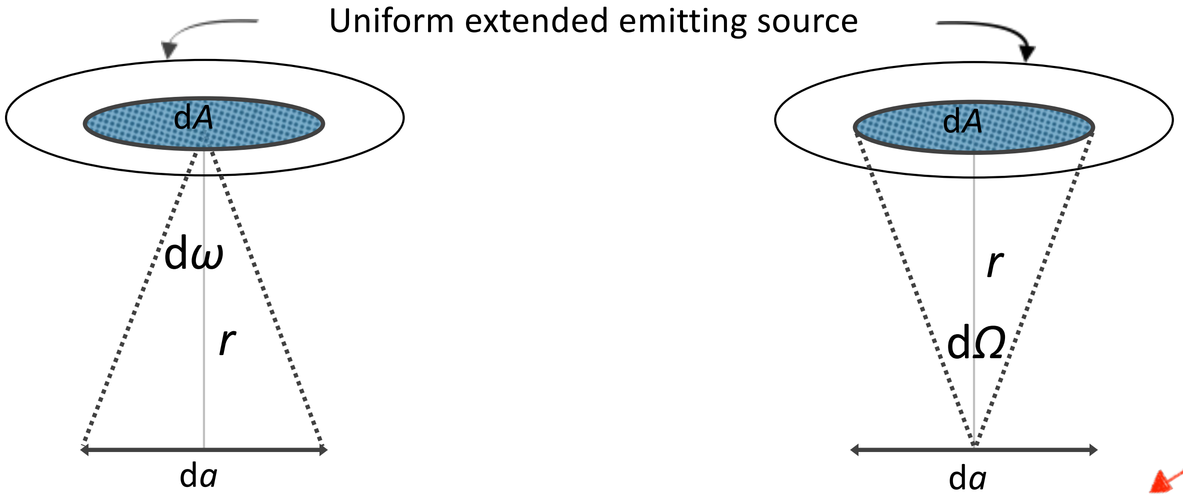
Surface brightness and flux density

- Brightness is a property of the source – it doesn’t change with distance*
- A bigger telescope doesn’t help in seeing lower brightnesses (in fact is often worse!) – the increase in area is offset by the smaller beam
- For extended sources (bigger than the telescope beam) think brightness

- Flux density depends on the distance to the source
- A bigger telescope (more area) can see lower flux densities
- For point sources (smaller than the telescope beam) think flux density

Equivalence of Specific Intensity & Surface Brightness

Emitting
from dA
to da



Receiving
from dA
with da

- $P_{\mathrm{em}} = \int \quad I_\nu(\theta,\phi)\cos\theta \mathrm{d}A\mathrm{d}\nu\mathrm{d}\omega$
- Since d$\omega = \mathrm{d}a/r^2$

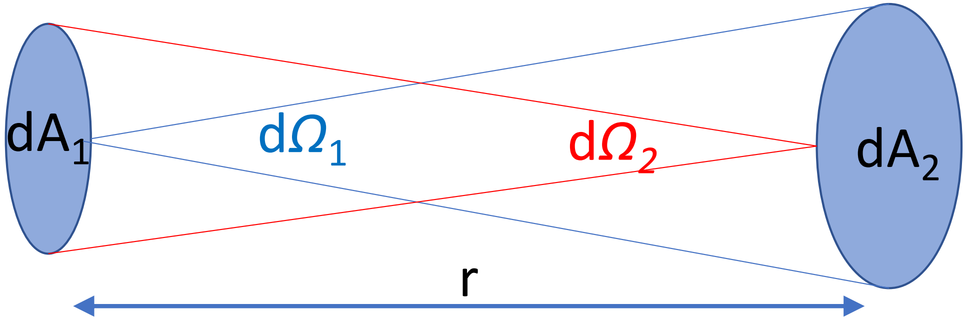
$$P_{\mathrm{em}} = \int \quad I_\nu(\theta,\phi)\frac{\cos\theta}{r^2}\mathrm{dAd}\nu\mathrm{da}$$

If $P_{\mathrm{em}} = P_{\mathrm{rec}}$
$I_\nu \;=\; B_\nu$

- $P_{\mathrm{rec}} = \int \quad B_\nu(\theta,\phi)\cos\theta \mathrm{d}a\mathrm{d}\nu\mathrm{d}\Omega$
- Since d$\Omega = \mathrm{d}A/r^2$

$$P_{\mathrm{rec}} = \int \quad B_\nu(\theta,\phi)\frac{\cos\theta}{r^2}\mathrm{dad}\nu\mathrm{d}A$$

Independence of intensity on distance



- Consider two areas (for simplicity taken as parallel disks placed far away from each other) in free space.
- Consider rays passing through both surfaces: how does I_ν compare?
- Energy conservation:
  power in the rays passing through both surfaces is identical.

$$I_{\nu,1} dA_1 d\nu d\Omega_1 = I_{\nu,2} dA_2 d\nu d\Omega_2$$
$$I_{\nu,1} dA_1 d\nu (dA_2/r^2) = I_{\nu,2} dA_2 d\nu \, (dA_1/r^2)$$
$$I_{\nu,1} = I_{\nu,2}$$

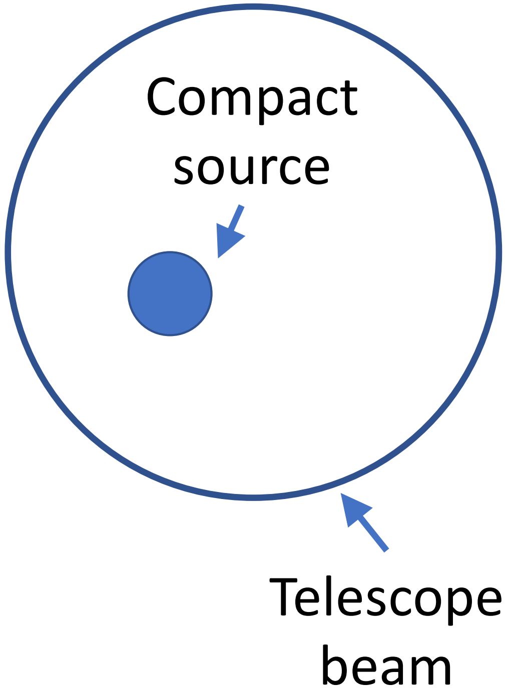
- Conclusion: Intensity is independent of distance.
- Also surface brightness is independent of distance.

Flux density and distance

- For the case of a compact source observed with a telescope whose beam solid angle is much larger than the source

  $F_{\nu} = \int \quad B_{\nu}(\theta, \phi)\cos\theta \mathrm{d}\Omega = B_{\nu}\Omega_{\mathrm{source}}$

  (other cases will be considered later in the course).
- If one moves a source further away, then the received flux decreases (with the inverse square law) because the solid angle subtended decreases, while the intensity remains constant.



Blackbody Radiation

The Planck function: $B_\nu(T) = \frac{2k_\mathrm{B}T\nu^2}{c^2}\left[\frac{h\nu/k_\mathrm{B}T}{\mathrm{e}^{h\nu/k_\mathrm{B}T}-1}\right] \quad \mathrm{Watts\,m^{-2}\,Hz^{-1}\,sr^{-1}}$

This formula describes a perfect **thermal** radiator or absorber.

The Rayleigh-Jeans Approximation

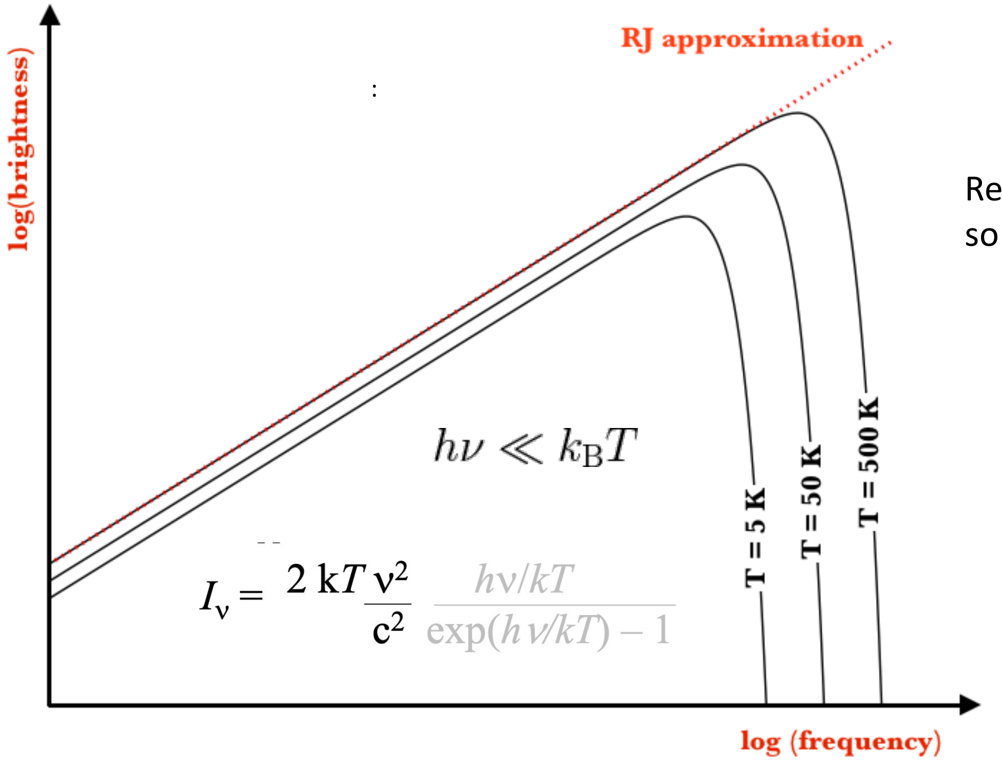
In the radio band we can use a nice approximation to the Planck function, because in general:

$$h\nu \ll k_\mathrm{B}T$$

In these circumstances we can use the RJ approximation:

$$B_\nu(T) = \frac{2k_\mathrm{B}T\nu^2}{c^2} \quad \mathrm{Watts\,m^{-2}\,Hz^{-1}\,sr^{-1}}$$

Blackbody Radiation



Remember we can sometimes use B and I

"Effective" Brightness Temperature

In using the **Planck formula** (or the **RJ approximation**) we've implicitly assumed that the source we're looking at is a perfect blackbody. In reality this will not be the case.

Most sources are not blackbodies - but it's convenient to use a blackbody as a reference.

We define a **brightness temperature**, T_B, which is equivalent to the temperature that a source would have, given its brightness, B_ν , ***if it were a blackbody***.

$$\Longrightarrow \quad \boldsymbol{T_{\mathrm{B}} = \frac{I_\nu \lambda^2}{2k_{\mathrm{B}}}}$$

Using this equivalence we find that astronomical sources have a wide range of brightness temperatures. **However…**

If a source is found to have a brightness temperature $\boldsymbol{T_B > 10^5}$ **K** then we know that it is **not a thermal source**. The radiation is coming from a **non-thermal particle population**.

Example: The Moon

Question: If the average temperature of the moon is approximately 210 K, what would its specific intensity be at a frequency of 1.4 GHz?

Solution: At 1.4 GHz and 210 K, $h\nu \ll k_B T$. So we can use the RJ approximation.

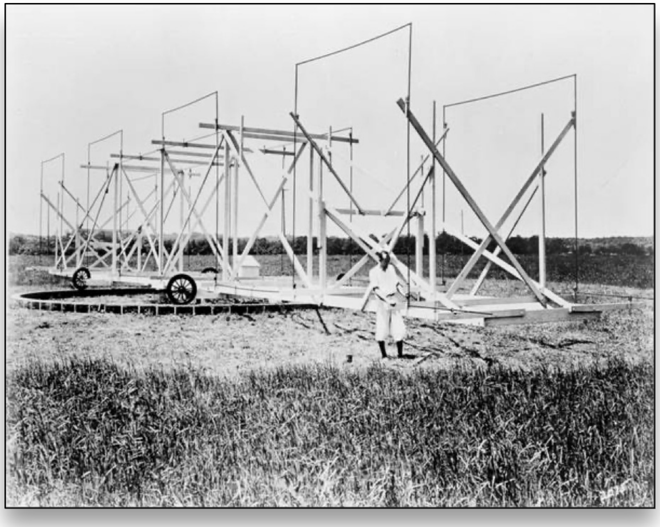
$$B_\nu(T) = \frac{2k_{\mathrm{B}}T\nu^2}{c^2} \quad \mathrm{Watts\,m^{-2}\,Hz^{-1}\,sr^{-1}}$$

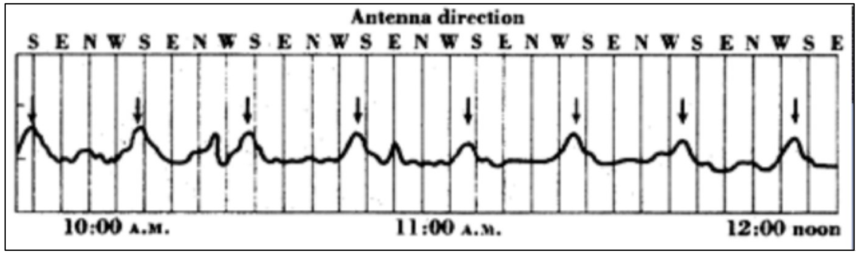
If we plug the numbers above into this equation, we find that
$B_\nu(210\ \mathrm{K}) = 1.32 \times 10^{-19}$ Watts $\mathrm{m^{-2}\ Hz^{-1}\ sr^{-1}}$

The Jansky unit for flux density

1 Jansky = 10^{-26} Watts m^{-2} Hz^{-1}

Named after Karl G. Jansky





Example: The Moon

Question: If the average temperature of the moon is approximately 210 K, what would its flux density in **Janskys** be at a frequency of 1.4 GHz?

Solution:

We found that at $B_\nu(210\ \mathrm{K}) = 1.32 \times 10^{-19}$ Watts $\mathrm{m}^{-2}\ \mathrm{Hz}^{-1}\ \mathrm{sr}^{-1} = 1.32 \times 10^{7}$ Jy sr^{-1}

To go from specific intensity to flux density, we need to multiply by the solid angle of the moon in steradians, which we had calculated to be 6×10^{-5} sr.

Its flux density is therefore 790 Jy

Linking Brightness, Intensity and Temperature

$$S = \int I_\nu \, d\Omega \longrightarrow S = I_\nu \Omega$$

Flux Density

Brightness

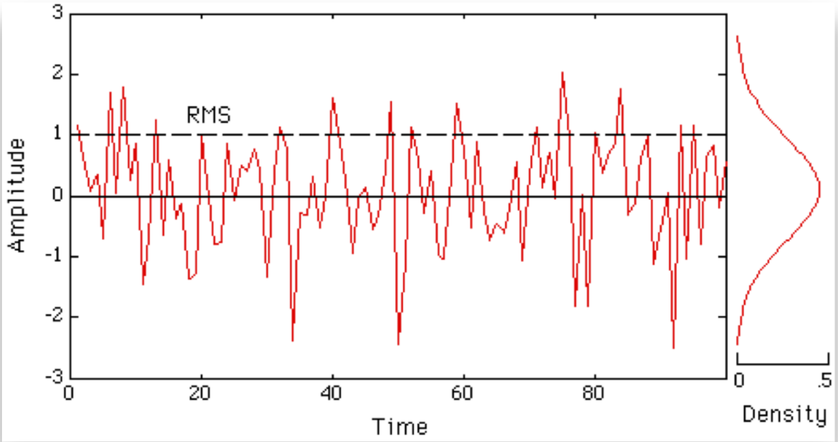
If brightness is constant over the solid angle

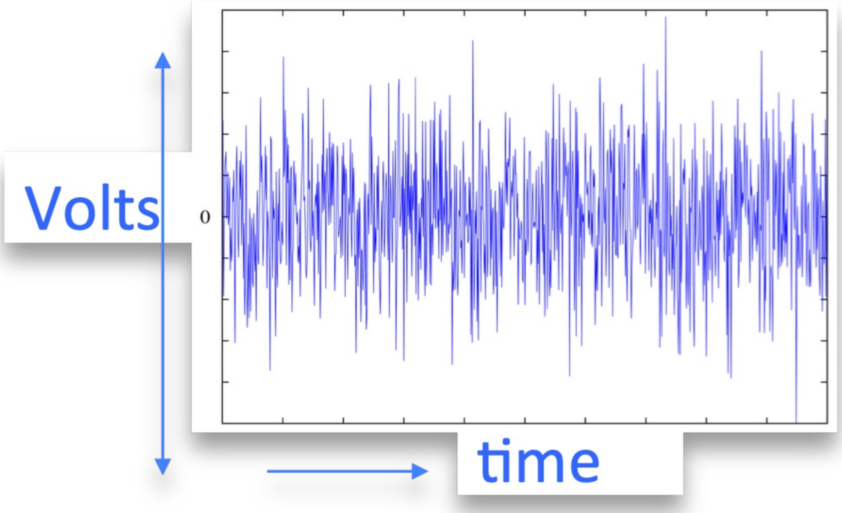
\frac

Using: $$\boldsymbol{T_{\mathrm{B}} = \frac{I_\nu \lambda^2}{2k_{\mathrm{B}}}}$$

$$\frac{S}{\Omega} = 2k_B \, T_B \, /\lambda^2$$

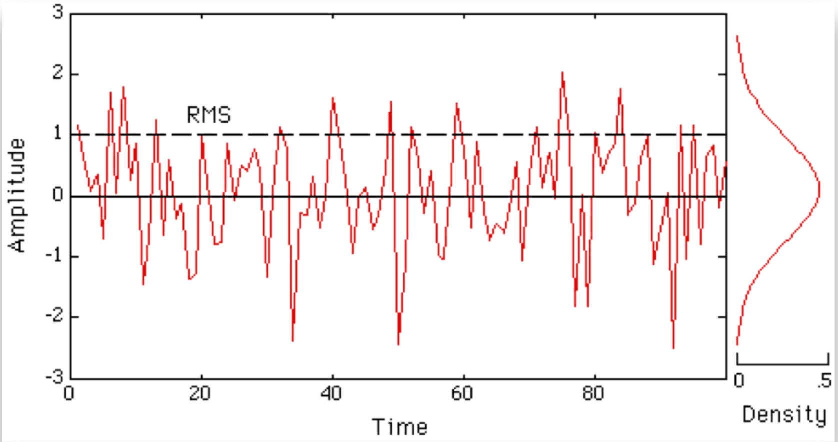
Random Noise

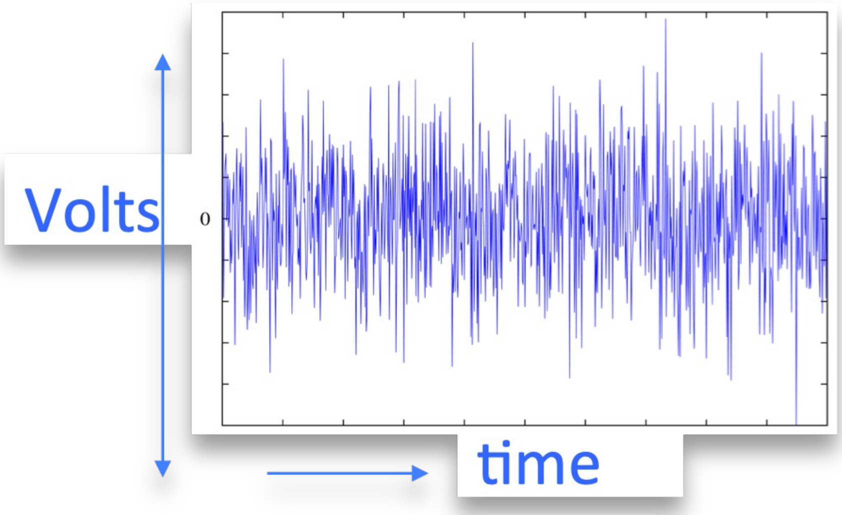




- ***All*** the natural signals with which we are dealing have been generated by *random processes* of one form or another. When incident on the receiver they all have the same basic form: random variations of the incident electric field in amplitude and phase.
- In the radio receiver the electric field variations are turned into random voltages with a **Gaussian probability distribution**

Random Noise

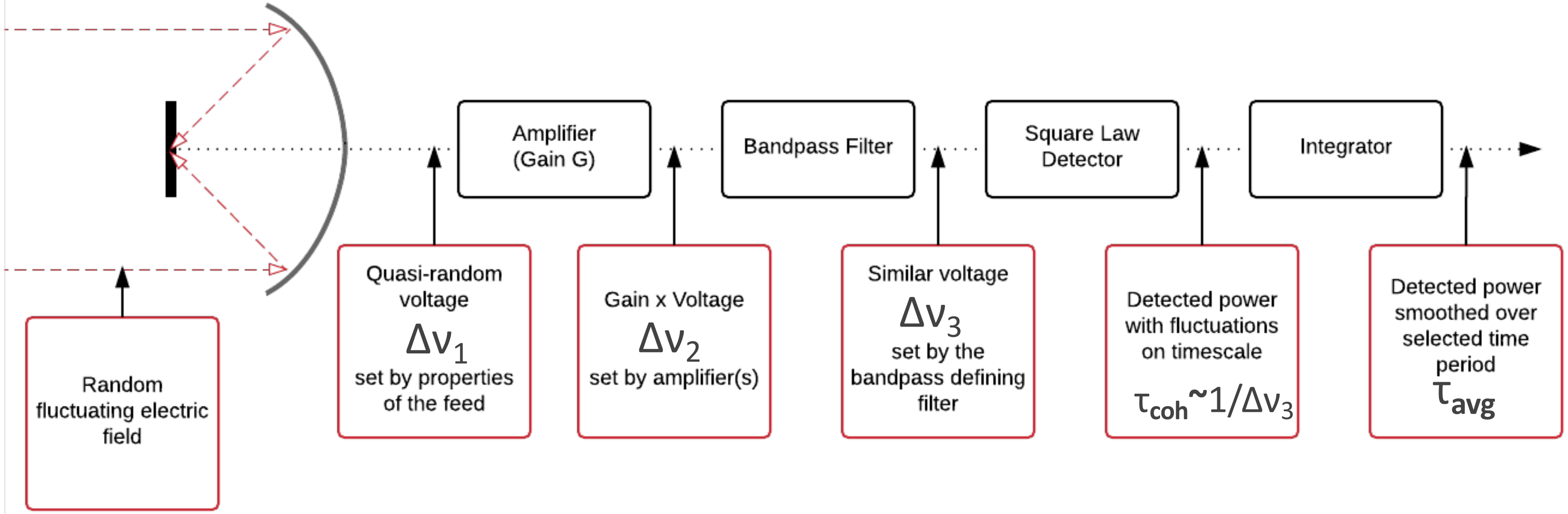




Signal voltages are indistinguishable from (and usually much smaller than) the voltages generated by thermal agitations in the resistive components of the receiver. These voltages are "white noise": the amount of power per unit bandwidth is independent of frequency (as in the Nyquist formula for resistor noise power = $k_B T$ W Hz^{-1}). The only stable, and hence useful to measure, quantity at a single point in the sky *is the average power* $<V^2> = \sigma^2$ (i.e. the "variance" of a random variable).

Basic Radiometer (receiver)

A basic receiver can be composed of a standard set of elements



Amplifier (Gain G)

Bandpass Filter

Square Law Detector

Integrator

Random fluctuating electric field

Quasi-random voltage $\Delta\nu_1$ set by properties of the feed

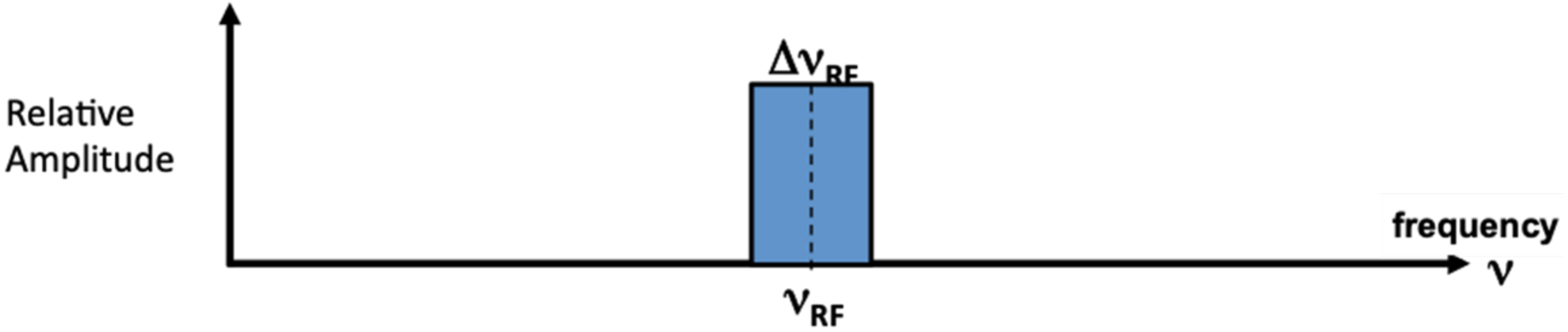
Gain x Voltage $\Delta\nu_2$ set by amplifier(s)

Similar voltage $\Delta\nu_3$ set by the bandpass defining filter

Detected power with fluctuations on timescale $\tau_{coh} \sim 1/\Delta\nu_3$

Detected power smoothed over selected time period τ_{avg}

Radio astronomy signals are VERY WEAK, so need a huge amount of amplification typically >100 dB before they can be measured and quantified.

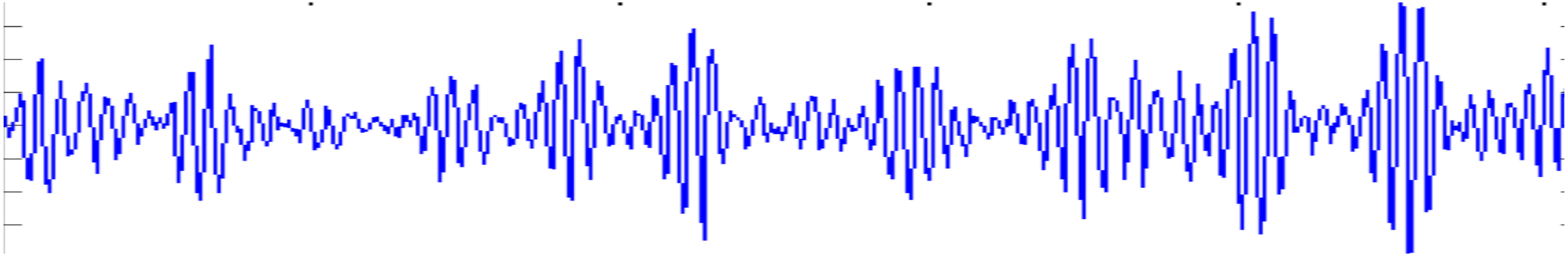


Restriction due to bandpass filter

Band-limited Noise

$$\tau_{\mathrm{coh}} = \frac{1}{\Delta\nu}$$

Band-limited Noise



- Noise frequency spectrum restricted by a band-pass filter covering a range of frequencies $\Delta\nu_{RF}$. The result is random noise, but now with some statistical structure imposed.
- Typical zero crossing times Δt_{RF} are separated by $1/\nu_{RF}$ and the amplitude modulation "envelope" exhibits a characteristic "coherence time" $\tau_{coh} \sim 1/\Delta\nu$. Within τ_{coh} the voltage is quasi-sinusoidal and hence has a quasi-predictable phase.
- Think of the signal as a sum of random sine waves and the concept of "beats". Modulation arises when two pure sine waves with frequencies ν_1 and ν_2 are added together. The resulting function in time has a modulation period ("beats") = $1/[(\nu_1 - \nu_2)]$.

Key concepts

1. Radio astronomy signals need a high degree of amplification before they can be measured/quantified

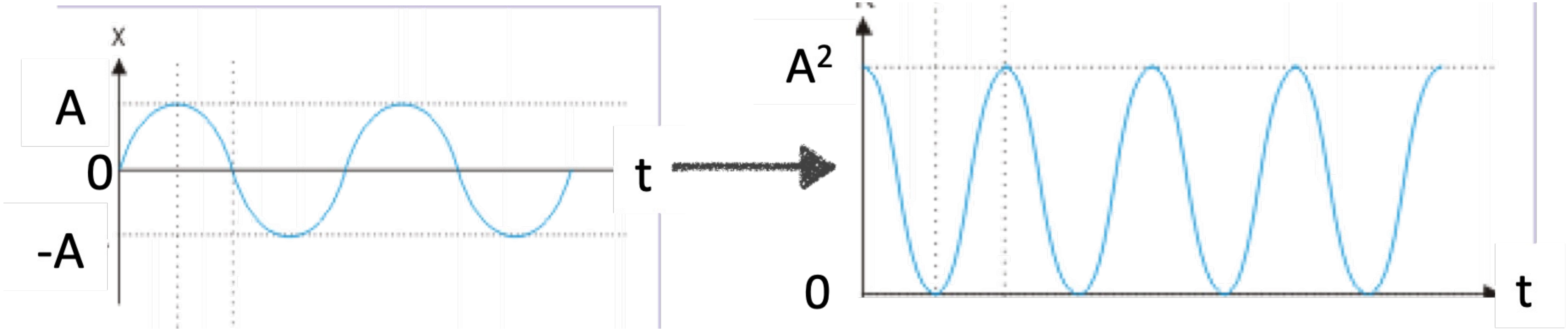
2. Noise *voltage* signals have zero mean and fluctuate on rapid timescales $1/\nu_{RF}$.

3. A ν_{RF} = 1 GHz means voltage varies on a 1 nanosecond (10^{-9} s) timescale, and envelope varies on timescales τ_{coh}~$1/\Delta\nu_{RF}$ ~ 10 nanoseconds for $\Delta\nu_{RF}$ = 100 MHz. This means the envelope typically would "contain" ~10 wavelengths of a quasi-sinusoid.

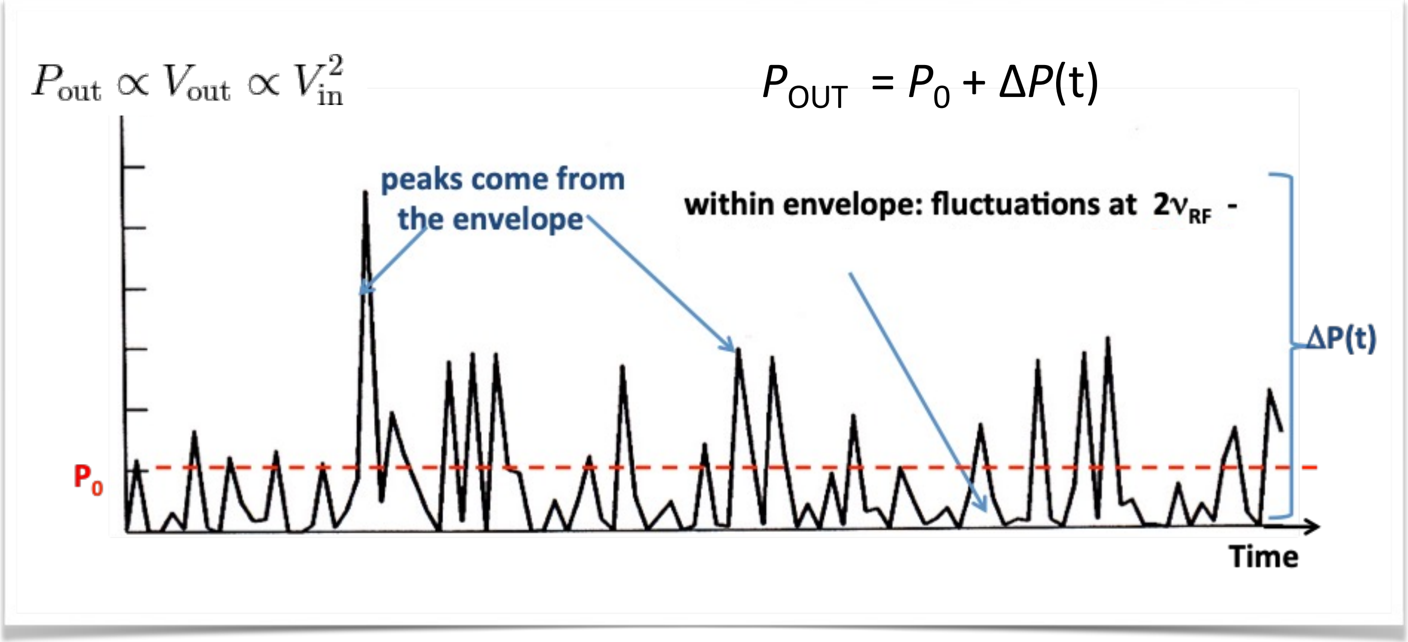
4. We need a steady non-zero signal, so we measure *power* and average for long periods.

5. Simplest way to measure power: put amplified voltage into a semiconductor "square law" detector – for *small input signals* $\mathbf{V_{OUT}}\ \alpha\ \mathbf{V_{IN}}^{2}$, i.e. proportional to the power in the input signal.

Square Law Detection (monochromatic example)



Fluctuating Power



- P_0 = stable component (what we want)
- ΔP(t) = strongly fluctuating component on timescales $\tau_{coh} = 2\times1/\Delta\nu_{RF}$ (what we need to average out)

Square Law Detection

- Square-law detector: multiplies a signal with itself.
- This is a **non-linear process** - the input frequencies mix together to give new frequencies.
- Even if the **input** signal has **Gaussian** statistics, the **output** signal has **non-Gaussian** statistics.

Central limit theorem: The sum of any large sample of independent random variables will exhibit a Gaussian distribution.

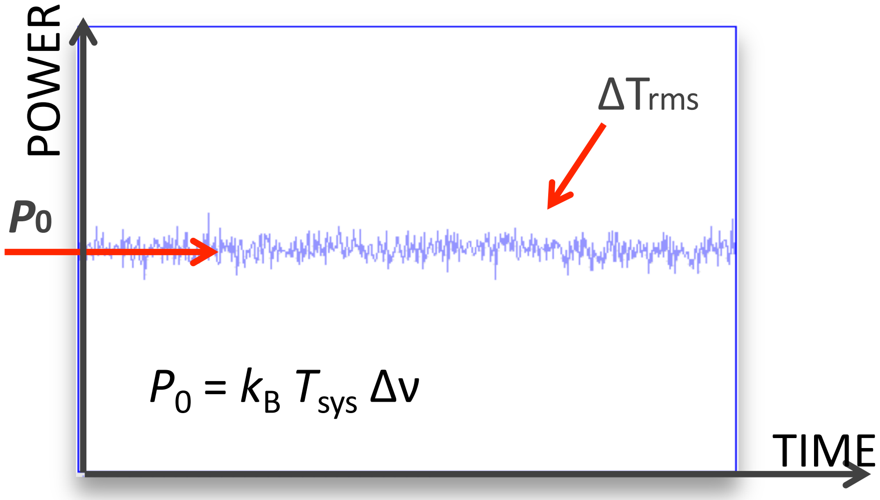
- Although $P(t) = [P_0 + \Delta P(t)]$ has a very non-Gaussian distribution, the averaged power $P_0 \pm \delta P(t)$, averaged over long τ_{avg}, **is** Gaussian and will have

$$\delta P_{\text{rms}} = \sqrt{2}\, P_0$$

After Square-Law Detection

RADIOMETER EQUATON:

$$\Delta T_{\mathrm{rms}} = \frac{T_{\mathrm{sys}}}{\sqrt{\Delta\nu\,\tau}}$$



With random noise signals ***all we can do*** is observe the fluctuating power output of the receiver over a long enough time τ_{avg} to obtain a result of the desired accuracy. Increasing the receiver bandwidth Δν also helps: the reduction in the level of fluctuations only goes as the square root of their product.

Natural signals are very weak and hence long integration times are often required for their detection.

Radiometer Equation

For an averaging time, τ_{avg}, the number of coherence times sampled is:

$$N_{\rm coh} = \frac{\tau_{\rm avg}}{\tau_{\rm coh}} = \tau_{\rm avg}\Delta\nu_{\rm RF}$$

To Nyquist sample: requires 2 samples per τ_{coh}

$$N_{\rm samp} = 2 \cdot \tau_{\rm avg} \cdot \Delta\nu_{\rm RF}$$

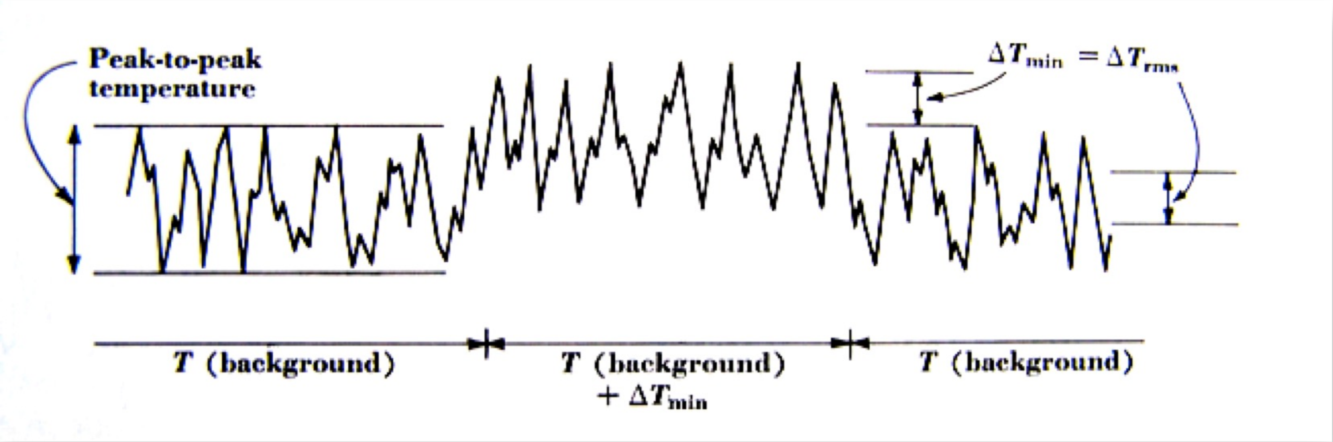
From the central limit theorem, the error on the mean is given by

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} \longleftarrow \sigma = \sqrt{2}\,P_0$$

and therefore:

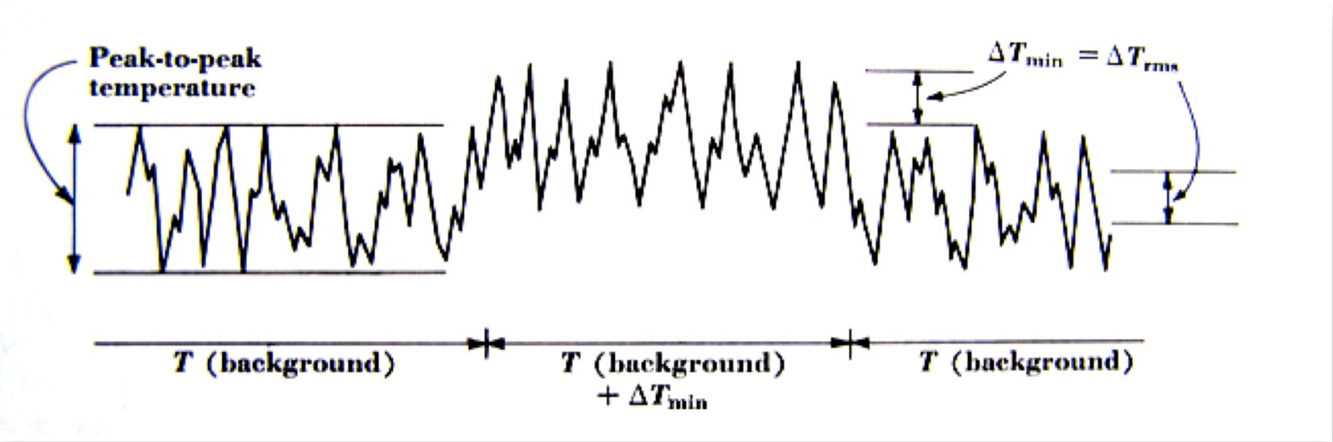
$$\delta P_{\rm rms} = \frac{\sqrt{2}\,P_0}{\sqrt{(2 \cdot \tau_{\rm avg} \cdot \Delta\nu_{\rm RF})}} \longrightarrow \boxed{\Delta T_{\rm rms} = \frac{T_{\rm sys}}{\sqrt{\Delta\nu\,\tau}}}$$

Detecting a weak source



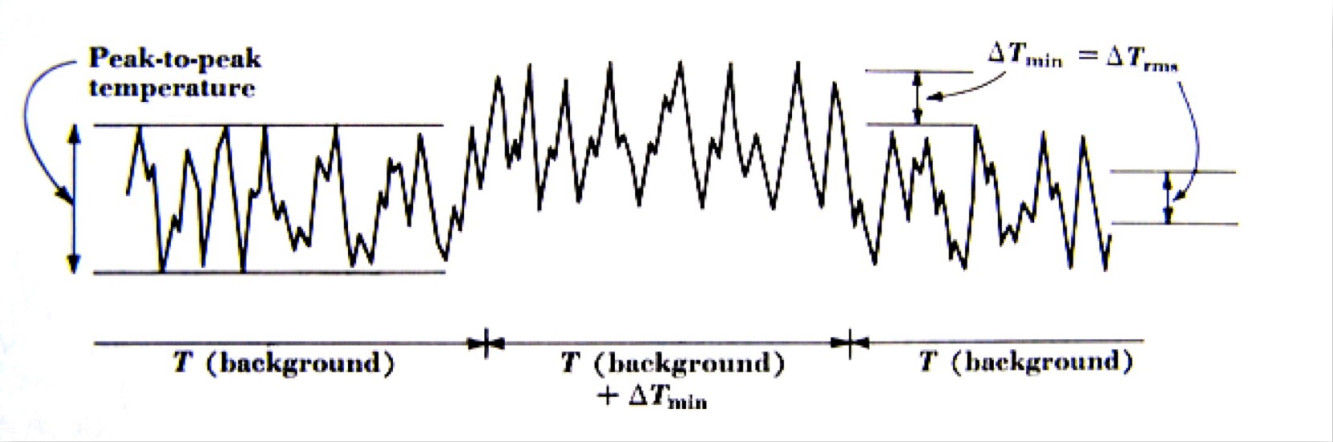
A schematic radio telescope record of the output power of a receiver after final averaging over a timescale of seconds as the beam moves over a weak source. The (slowly-varying) component of T_{sys} (due mainly to T_{LNA}) has been suppressed to emphasize the fluctuations. On the left and right the telescope "sees" only the background brightness temperature. In the central region the beam is pointed at a weak source comparable to ΔT_{rms} (~1 sigma for the chosen τ_{avg}). Because the telescope is pointed at the source for a time longer than τavg the rise in mean temperature due to the source can be detected.

Detecting a weak source



Example: for a 1 sigma “detection” of an antenna temperature T_A= 1 mK = ΔT_{min} with a total system temperature T_{sys} = 100 K and $\Delta\nu_{RF}$ = 100 MHz what τ_{avg} is needed. What about to increase this to 5 sigma.

Detecting a weak source



Example: for a 1 sigma "detection" of an antenna temperature T_A= 1 mK = $\Delta T_{\rm min}$ with a total system temperature $T_{\rm sys}$ = 100 K and $\Delta\nu_{\rm RF}$ = 100 MHz requires $\tau_{\rm avg}$ ~ 100 s. To increase this to 5 sigma requires an integration time of 2500 s *for the detection of weak sources* the *system performance must stay stable for long periods.*

Receiver Temperature: Hot & Cold Loads

How to measure T_{rec}: the (often dominating) noise contribution of the receiver to the T_{sys}? Subject the system to known sources of input power. If comparable (or larger) than associated with T_{rec}, only short integrations are needed to overcome the output fluctuations.

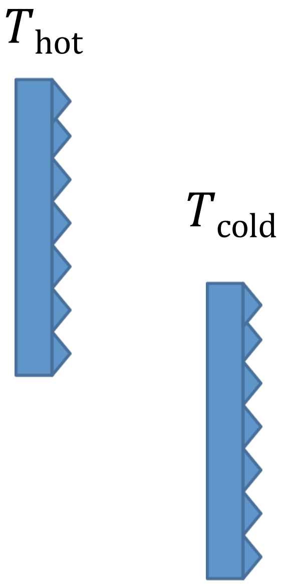
Eccosorb

Liquid nitrogen

Receiver Temperature: Hot & Cold Loads

Stick a black body emitter at two different temperatures in front of the receiver, e.g.

$$T_{\rm hot} \simeq 293\ {\rm K}\quad ({\rm room\ temp.})\qquad T_{\rm cold} \simeq 77\ {\rm K}\quad ({\rm Nitrogen})$$



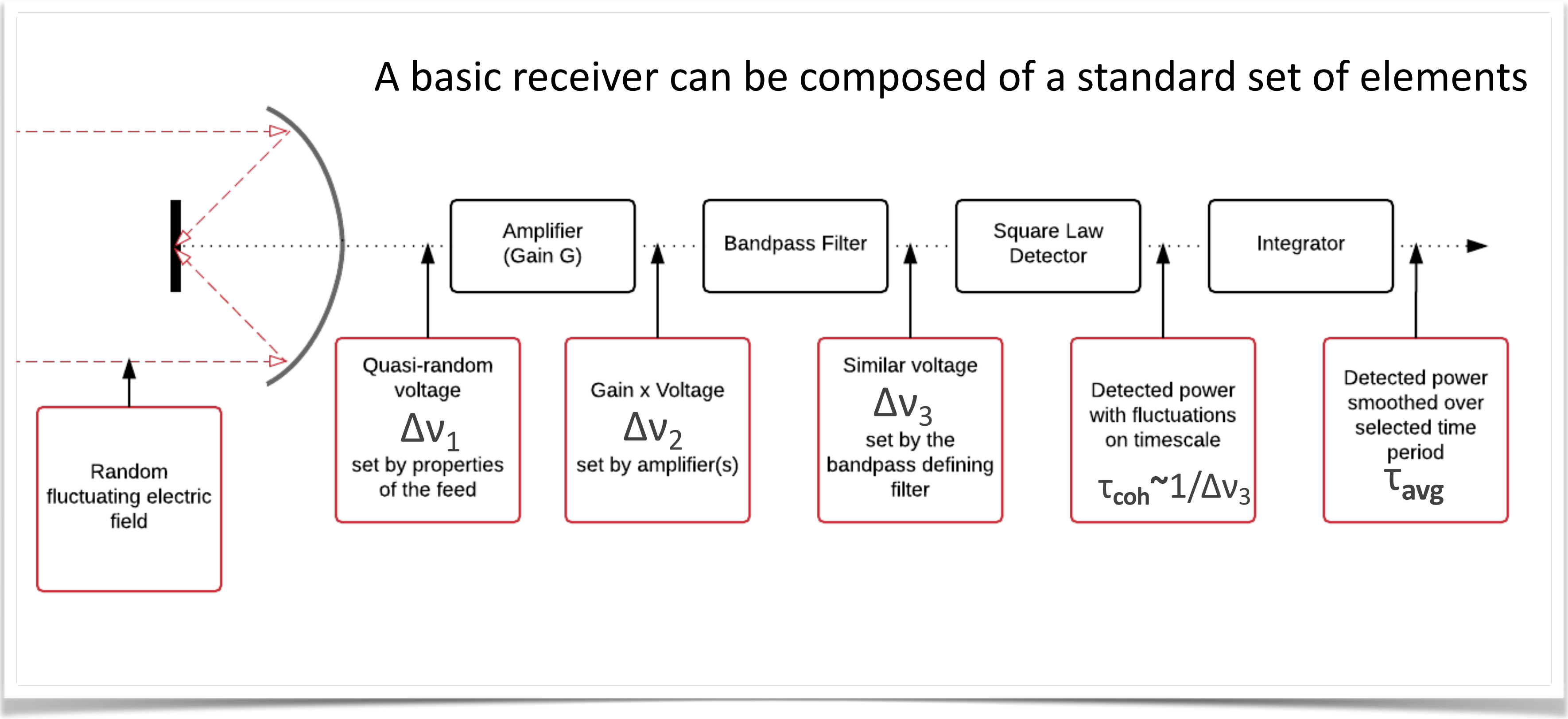
$$P_{\rm out,hot} \propto ({\rm Receiver\ Power\ Gain}) \times (T_{\rm rec} + T_{\rm hot})$$
$$P_{\rm out,cold} \propto ({\rm Receiver\ Power\ Gain}) \times (T_{\rm rec} + T_{\rm cold})$$

$$R = \frac{P_{\rm out,hot}}{P_{\rm out,cold}} = \frac{T_{\rm rec} + T_{\rm hot}}{T_{\rm rec} + T_{\rm cold}}$$

Measure ratio of output power

$$T_{\rm rec} = \frac{[T_{\rm hot} - R \cdot T_{\rm cold}]}{(R-1)}$$

Basic Radiometer (receiver)



A basic receiver can be composed of a standard set of elements

Amplifier (Gain G)

Bandpass Filter

Square Law Detector

Integrator

Random fluctuating electric field

Quasi-random voltage $\Delta\nu_1$ set by properties of the feed

Gain x Voltage $\Delta\nu_2$ set by amplifier(s)

Similar voltage $\Delta\nu_3$ set by the bandpass defining filter

Detected power with fluctuations on timescale τ_{coh}~$1/\Delta\nu_3$

Detected power smoothed over selected time period τ_{avg}

Summary

- Radio is the part of the E-M spectrum dominated by wave rather than photon behaviour.
- Antennas collect radiation from a region of sky defined by their aperture in wavelengths
- Flux density = power per square metre per Hz of bandwidth
- Brightness = Flux density per solid angle
- Brightness temperature: pretend the brightness you observe is from a Rayleigh-Jeans spectrum (even if it isn’t)
- Noise arises from everything between the telescope and the source, plus the antenna and electronics. Losses before the first amplifier matter a lot!
- White noise averages down as the square root of the number of data samples (bandwidth x time).
- Gain instabilities can limit sensitivity at low frequency=long integration times.