

# Introduction to Pulsar Timing

Dr Tom Scragg

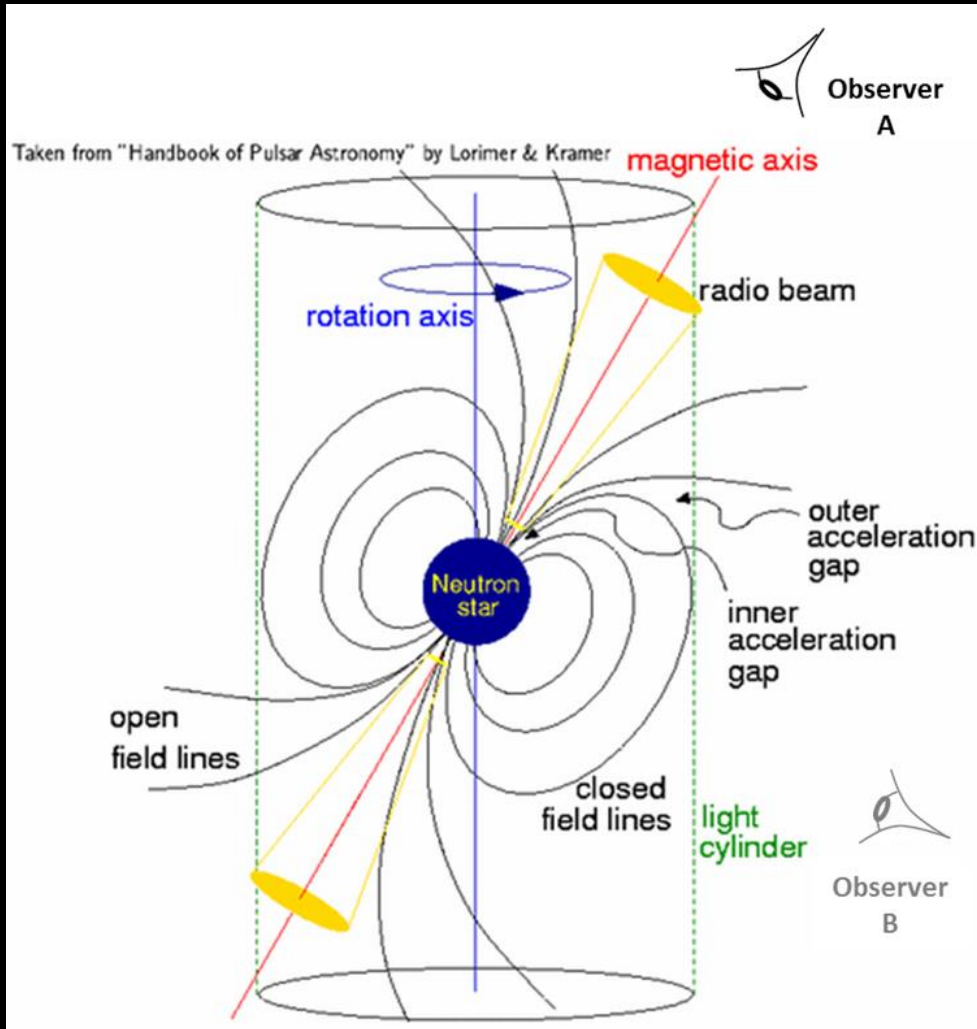
13/11/2024

DARA / HartRAO

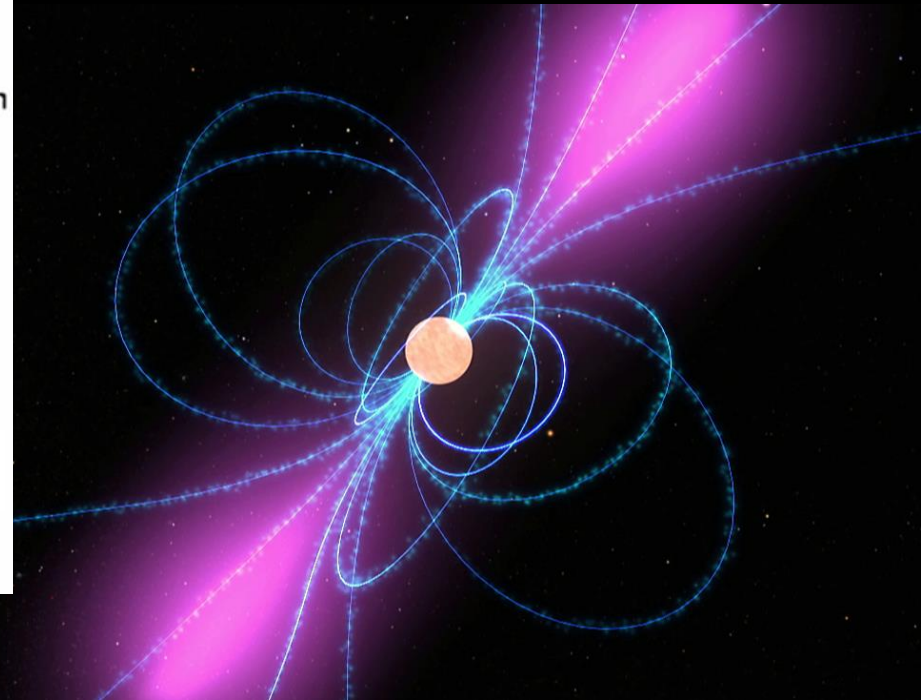


Image credit: NASA

# What are pulsars?



They are **not** stars that pulse.  
A pulsar is a neutron star where the magnetic pole is not aligned with the axis of rotation and generates a beam of radiation that periodically aligns with our line of sight



# Physics with Pulsars

## Time of arrival at different frequencies

Frequency dispersion along the line of sight gives us information on the neutral hydrogen / electron number density of the ISM

## By looking at the pulse profile

Changes in flux or phase or shape, we can derive structure in the emitting regions

## Pulsar timing

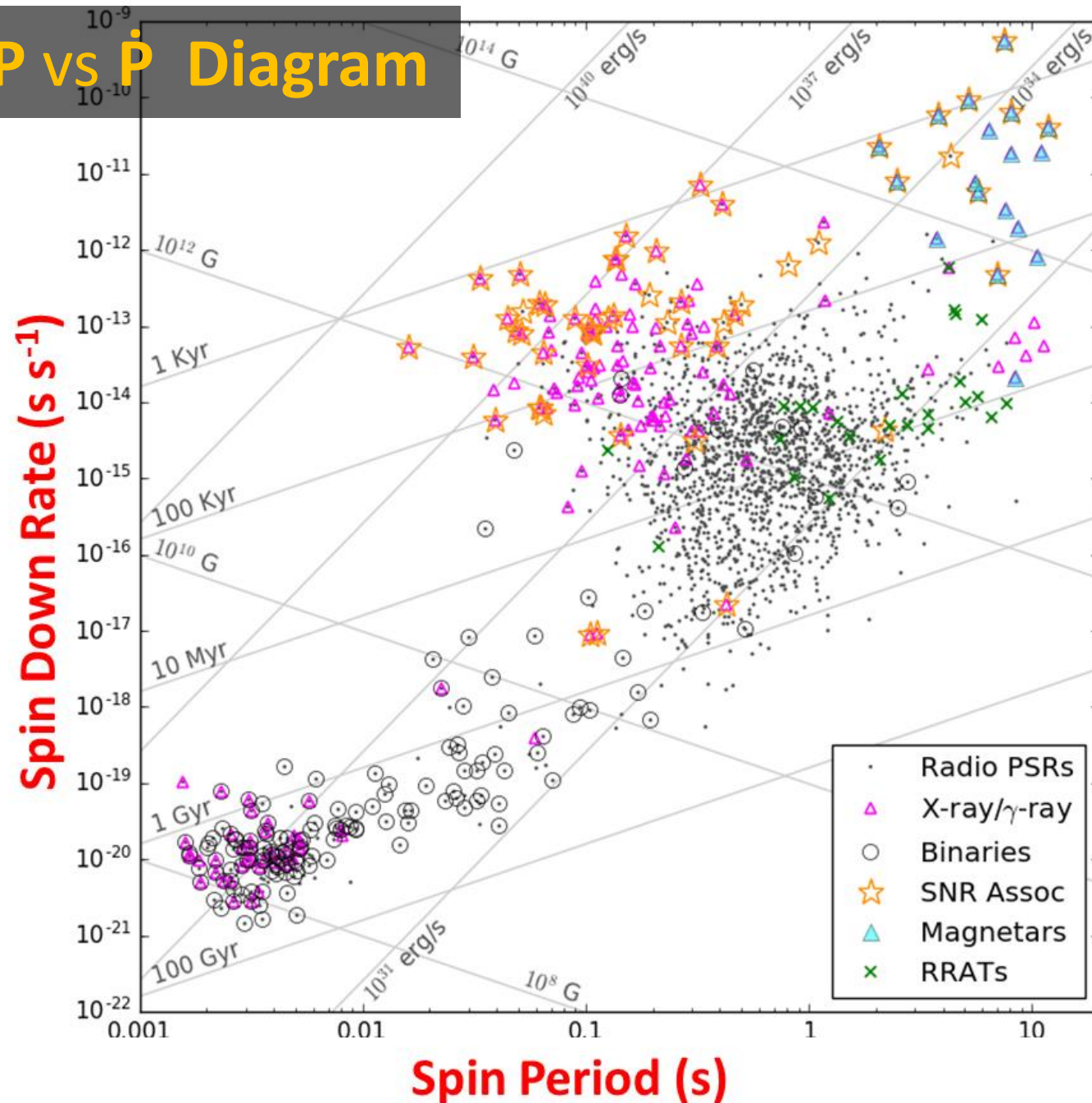
Spin down rate, RRATs & Nulling emission modes, Glitches (neutron star interiors)

## Pulsars in Binary Systems

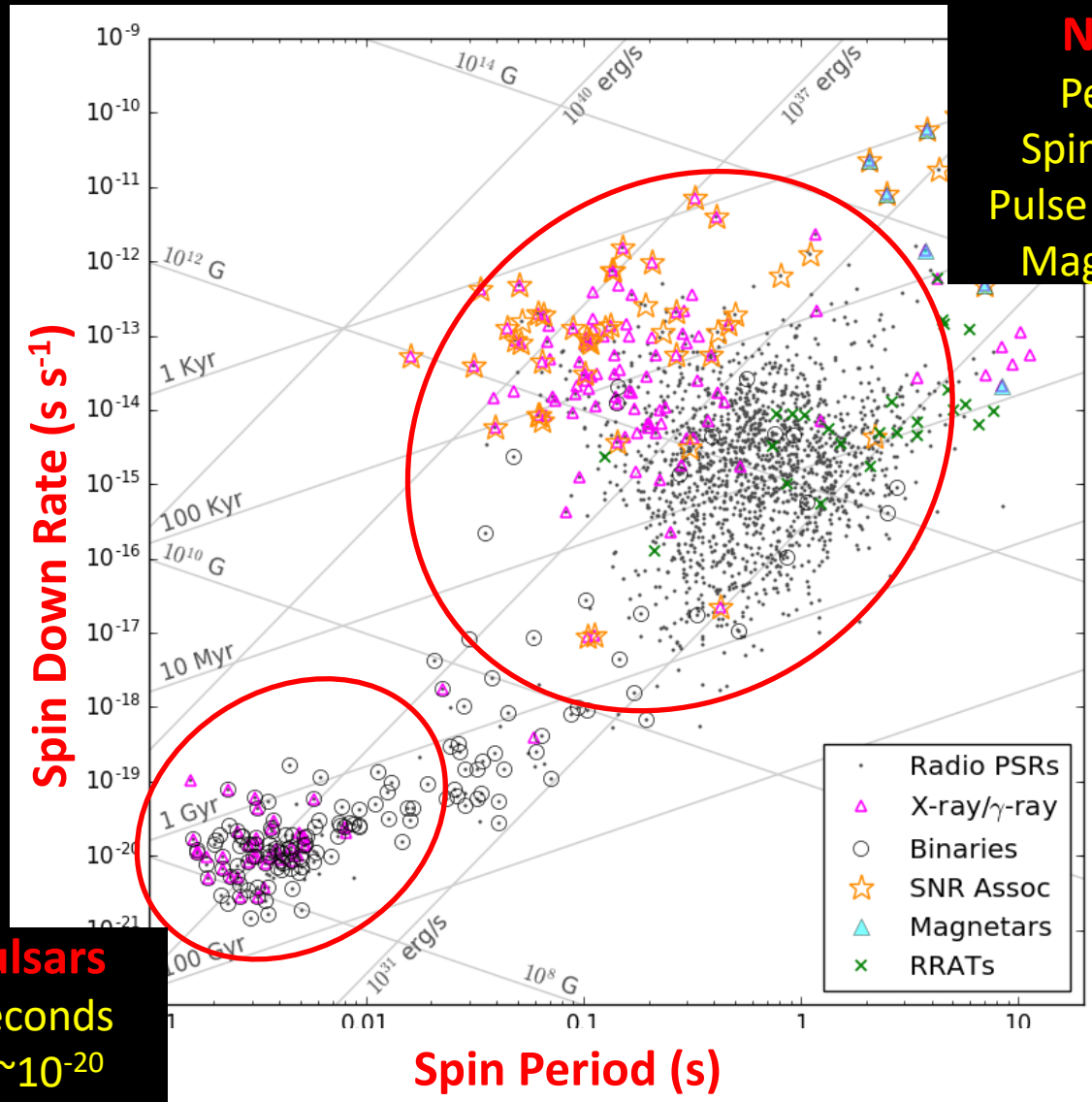
Interactions between companions ( Recycled pulsars / MSPs, Redbacks & Black Widows)

Post Keplerian Orbital mechanics, tests of General Relativity,  
Constrain theories on gravity waves

# Pulsar $\dot{P}$ vs $P$ Diagram



# Two Main Populations



**Normal pulsars**

Period  $\sim$  seconds

Spin down rate  $\sim 10^{-15}$

Pulse widths 100's of ms

Magnetic fields  $10^{12}\text{G}$

**Millisecond pulsars**

Periods  $\sim$  milliseconds

Spin down rate  $\sim 10^{-20}$

Pulse widths  $\sim \mu\text{s}$

Magnetic fields  $10^8\text{G}$

Image Credit:  
JBCA Internal Plot

# Timing of Pulsars

Regular monitoring of the rotation of a neutron star by noting the time of arrival of radio pulses.

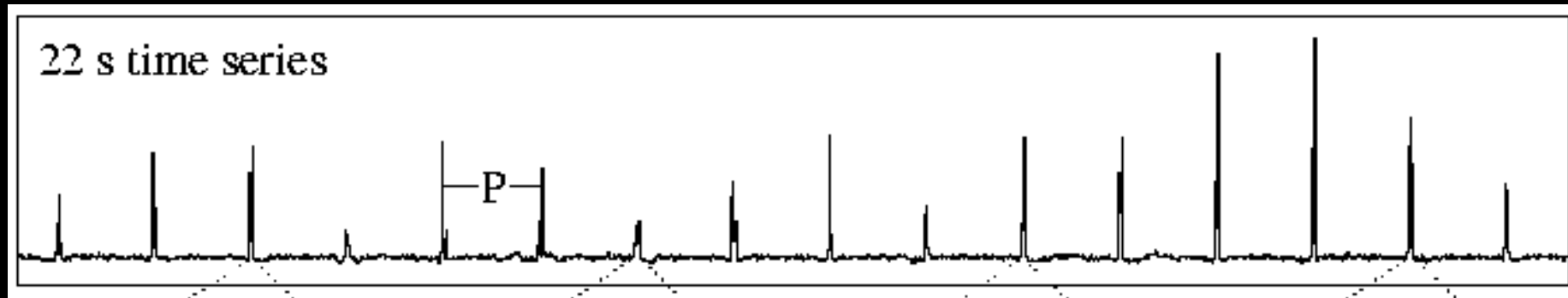


Image Credit: Adapted from Pulsar Astronomy

**P is the period between peaks (or a prominent feature on the pulse)**



# Ways of Measuring the Period

**Pick a point on the pulse and measure the time delay to the next occurrence**

- Simple case: 1 second period, 10% duty cycle, assume our accuracy of the pulse peak is 10% = 0.1s (100ms).

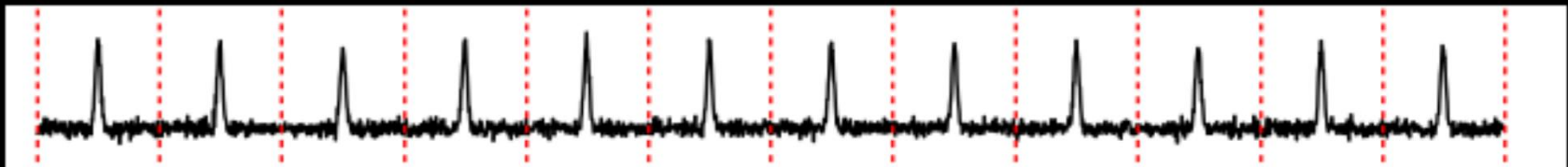


Image Credit: Scott Ransom

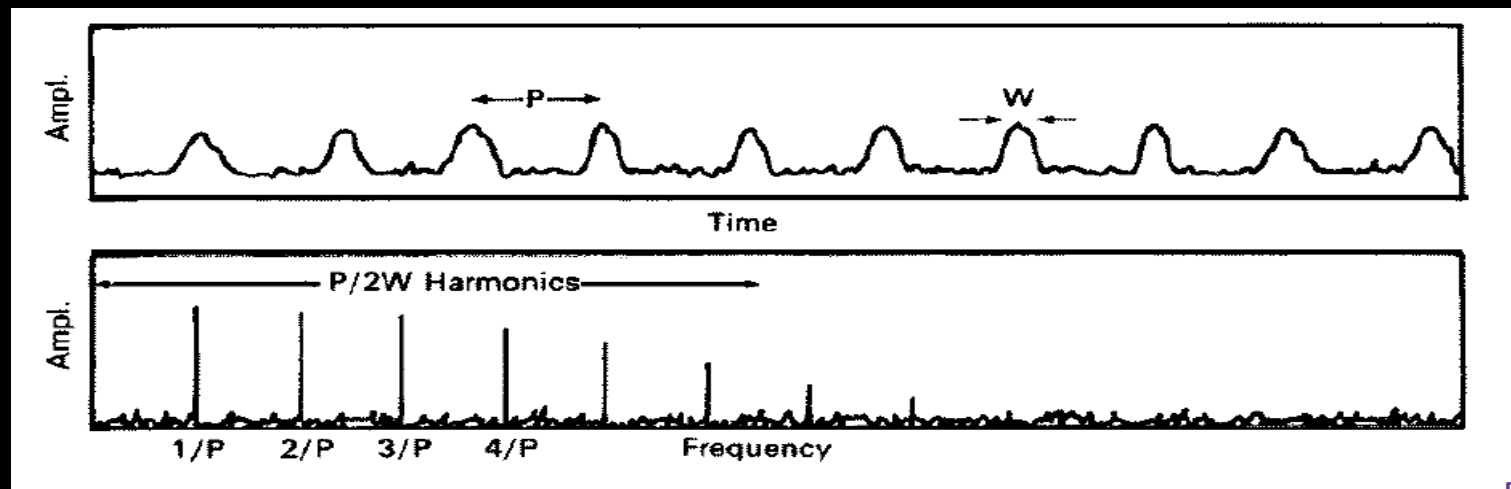
- Measure over  $n$  pulse periods and we can reduce the error in our estimation of the period by  $1/n$

**PSR J1737+0747 (a millisecond pulsar)**

$$P = 4.570136529159926 \text{ ms} \pm 0.1 \times 10^{-8} \text{ ns}$$

# Search: Periodogram & FFT Analysis

- **A data stream of  $N$  samples is folded at  $N$  different values of period  $P$ .**
  - If  $N = 10,000$  and we fold in steps of 1 sample period (1,2,3 ...100..1000) we have over 90,000 profiles to search for a pulse
  - Each profile will have a different value for improved SNR because the number of samples in each profiles is different.
- **FFT of the data sample**
  - Take the frequency transform of the time series data and looking for frequency components:
  - The width of the frequency peak,  $\Delta f$  is proportional to  $1/T$  ( $T$ = length of the data series in time). So for a 5 minute observation  $1/T \sim 3\text{ms}$  for a 1Hz pulsar)





# Integrated Pulse Profile

- **Pulses may be below the noise floor of our instrument**
  - Sum many observations to improve signal to noise
- **Pulses are not simple consistent shapes**
  - Profile Stabilisation Time: few 100's to a many 1000s of pulse periods.

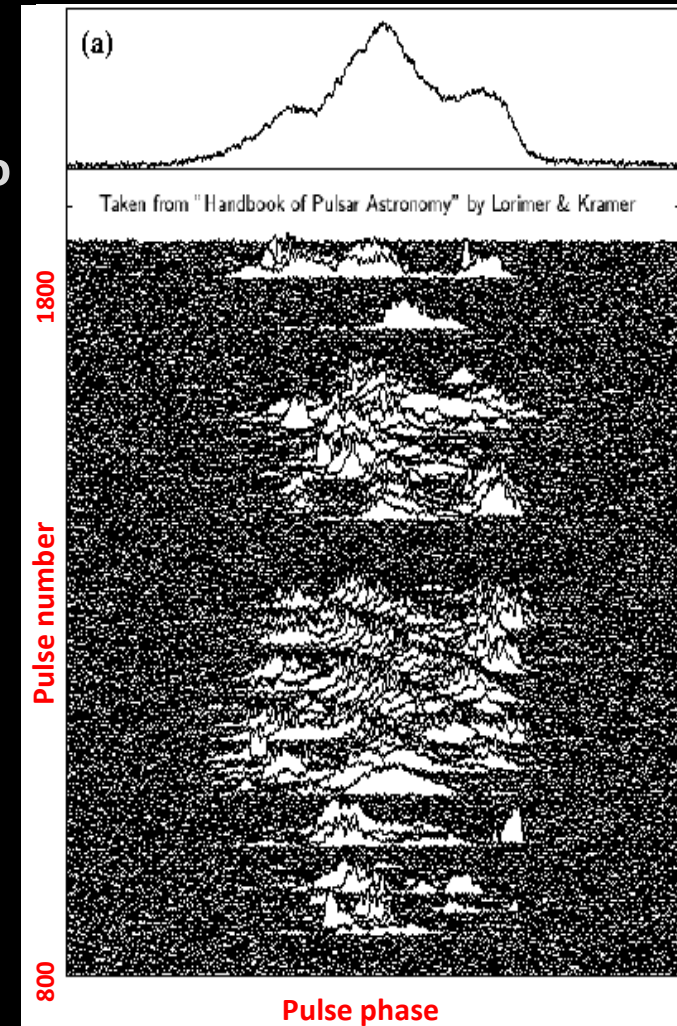
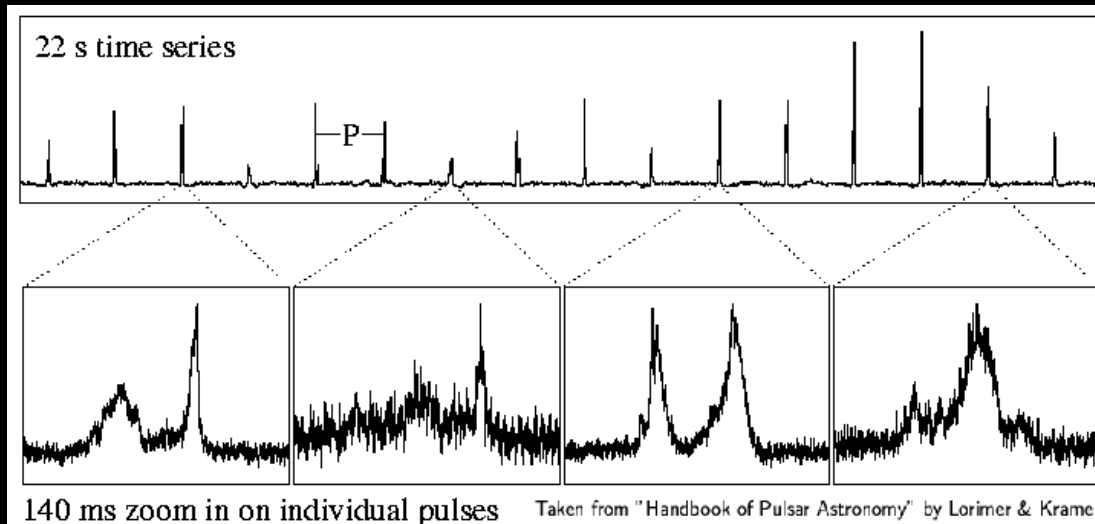
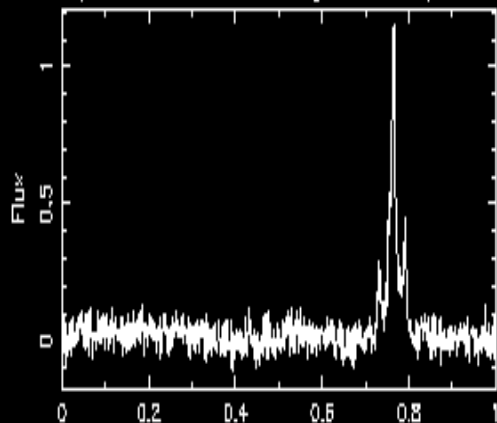


Image Credit: Adapted from  
Handbook of Pulsar Astronomy

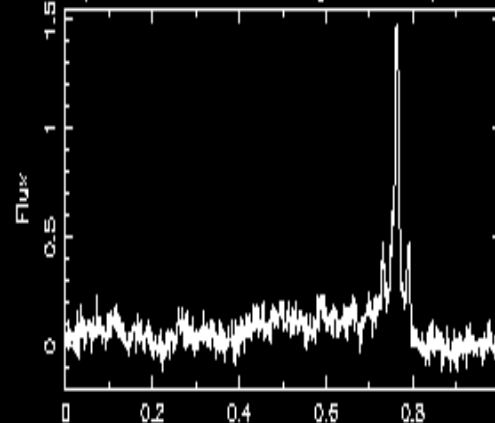
# Profile changes over Time

examples: Thesis T W Scragg 2020 JBCA, University of Manchester

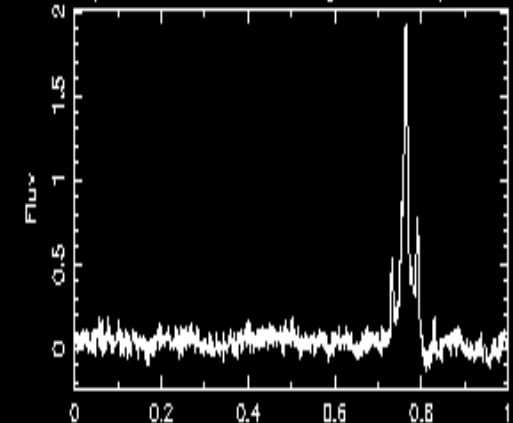
B0329+54 J0332+5434\_Da1478\_utc171027-1244\_8bDPF1.zap  
Freq: 1475.284 MHz BW: -64 Length: 328.000 S/N: 38.202



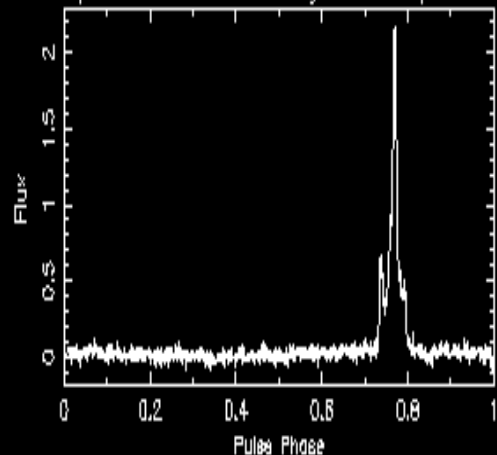
B0329+54 J0332+5434\_Da1542\_utc171027-1244\_8bDPF1.zap  
Freq: 1542.782 MHz BW: -64 Length: 328.000 S/N: 69.683



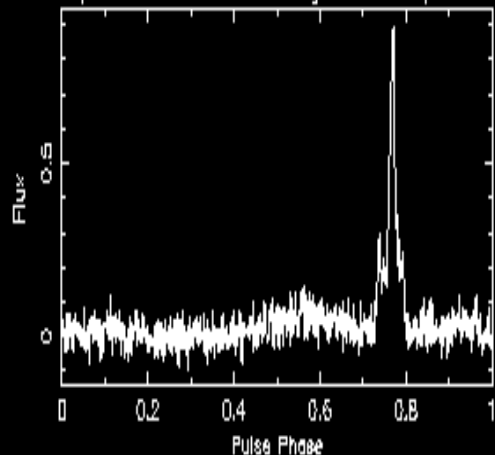
B0329+54 J0332+5434\_Da1478\_utc171027-1244\_8bDPF1.zap  
Freq: 1478.247 MHz BW: -64 Length: 328.000 S/N: 44.263



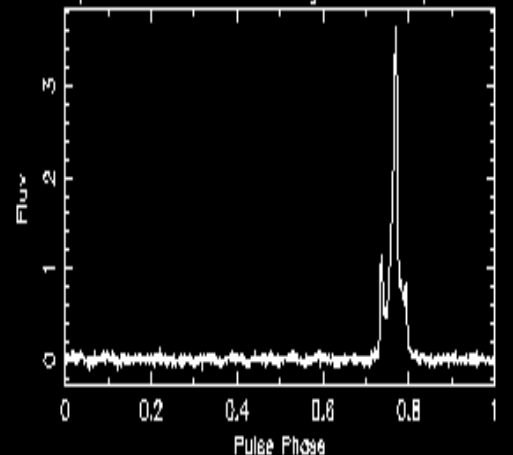
B0329+54 J0332+5434\_Da1478\_utc171027-1334\_8bDPF1.zap  
Freq: 1476.472 MHz BW: -64 Length: 321.000 S/N: 116.723



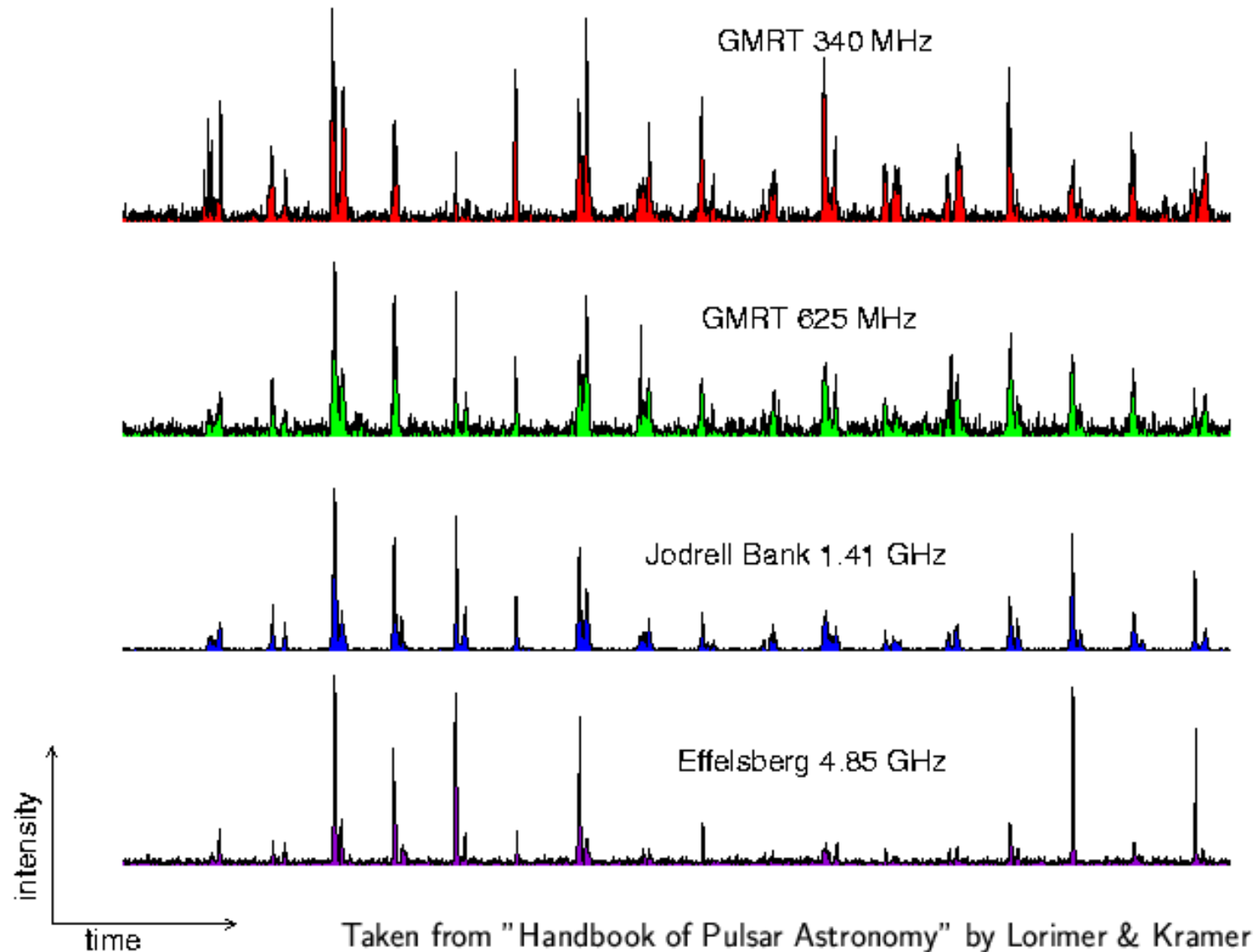
B0329+54 J0332+5434\_Da1542\_utc171027-1334\_8bDPF1.zap  
Freq: 1542.717 MHz BW: -64 Length: 321.000 S/N: 52.641



B0329+54 J0332+5434\_Da1478\_utc171027-1334\_8bDPF1.zap  
Freq: 1478.276 MHz BW: -64 Length: 321.000 S/N: 300.731

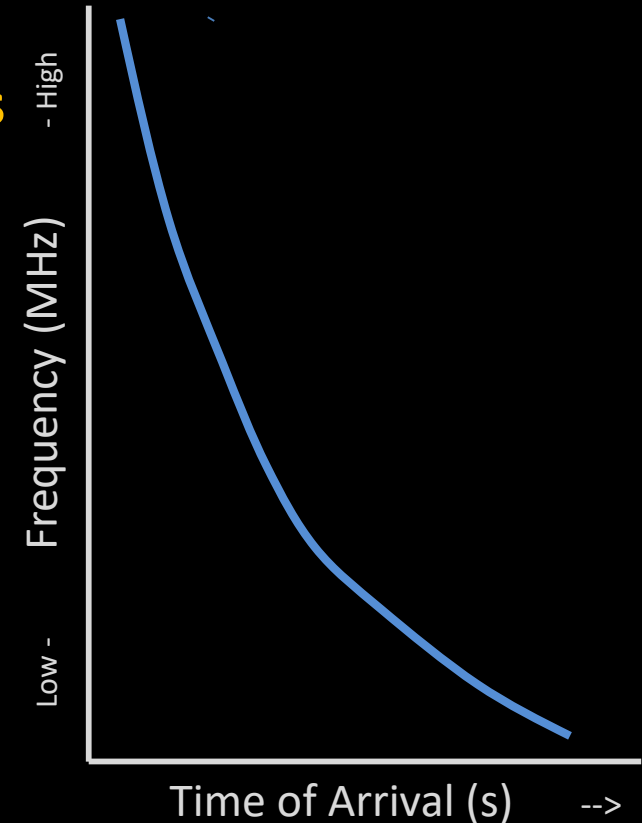


# Pulse Shapes at Different Frequencies



# Dispersion - 1

- The transmission of a radio signal through the interstellar medium (ISM) is delayed proportional to it's frequency
  - High frequencies arrive earlier than lower frequencies
- **Dedispersion**
  - We process the data to reverse the effect of the Interstellar Medium on the pulsar signal.
- **Dispersion Measure**
  - A measure of the electron density along the line of sight to the pulsar ( $\text{cm}^{-3} \text{ pc}$ )
  - Above 300 can be considered extragalactic



# Dispersion - 2

$\Delta t$  = time delay

$v_g$  = group velocity through a plasma

$D$  = Dispersion Constant

$DM$  = Dispersion Measure

$e$  = charge on the electron

$m_e$  = mass of the electron

$$t = \left( \int_0^d \frac{dl}{v_g} \right) - \frac{d}{c} \quad s \quad (a)$$

$$\Delta t = D \times (f_1^{-2} - f_2^{-2}) \times DM \quad s \quad (b)$$

$$D = \frac{e^2}{2\pi m_e c} \approx 4.148808 \pm 0.000003 \times 10^3 \text{ MHz}^2 \text{ pc}^{-1} \text{ cm}^3 \text{ s} \quad (c)$$

$$DM = \int_0^d n_e dl \quad \text{cm}^{-3} \text{ pc} \quad (d)$$

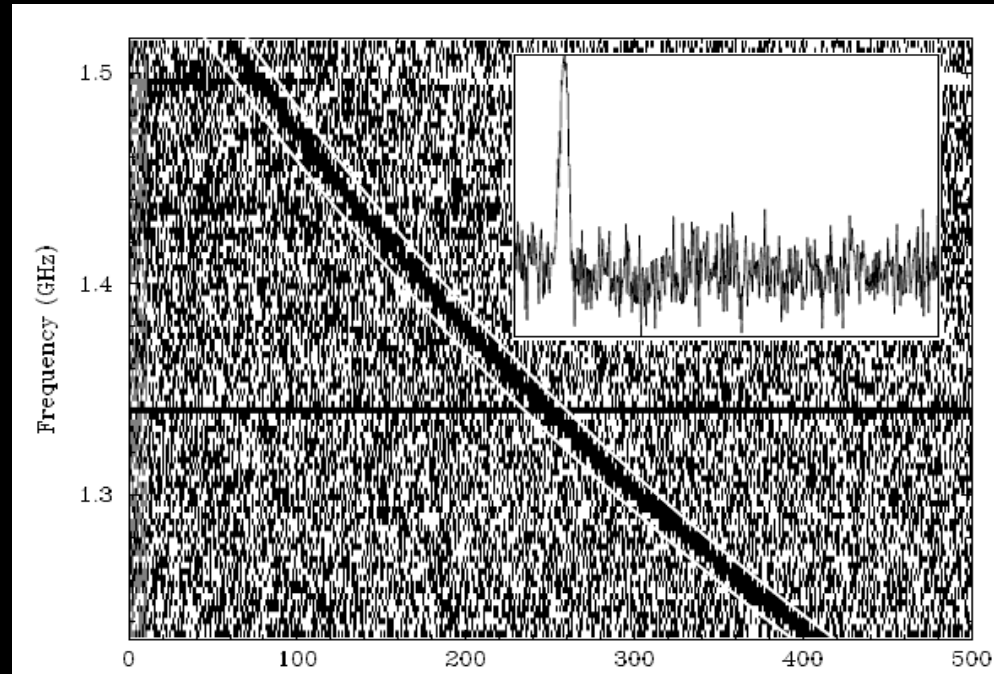
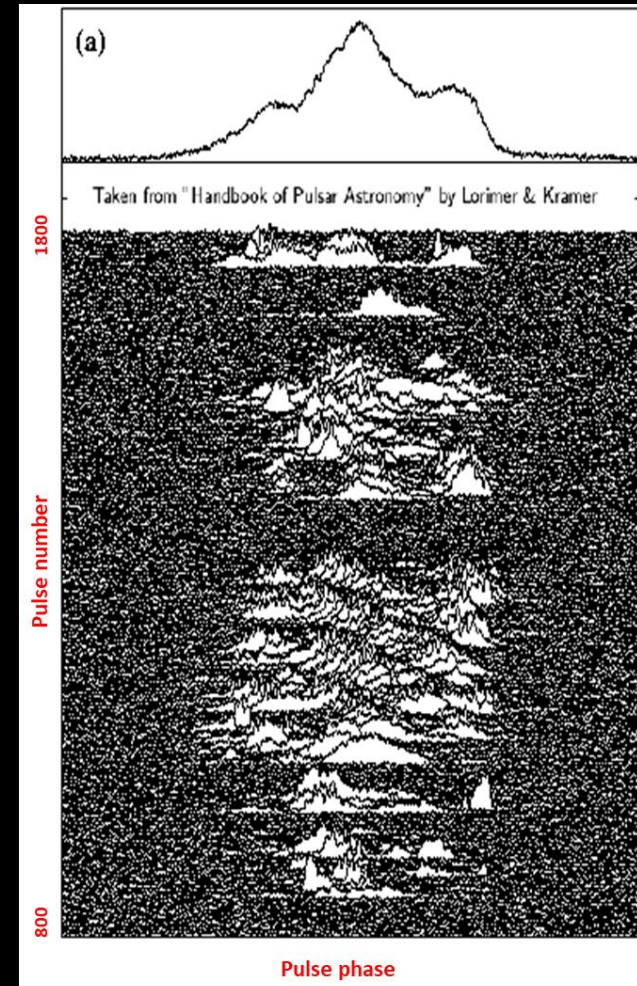


Image Credit: Handbook of Pulsar Astronomy  
Lorimer & Kramer

# Scrunching and Folding

- The noise within a pulsar signal is not correlated with respect to frequency or time.
- Scrunching in frequency
  - Filter banks separate the incoming signal into frequency channels
  - After dedispersion the signal is correlated wrt to time but the noise is not
  - Combining the separate frequency channels increases the signal and reduces the noise S/N.
- Folding in time:
  - We observe the pulsar for a much longer time than a single period of rotation.
  - Adding multiple periods together reduces the noise and improves signal gain (S/N)
  - *And generates a stable pulse profile*





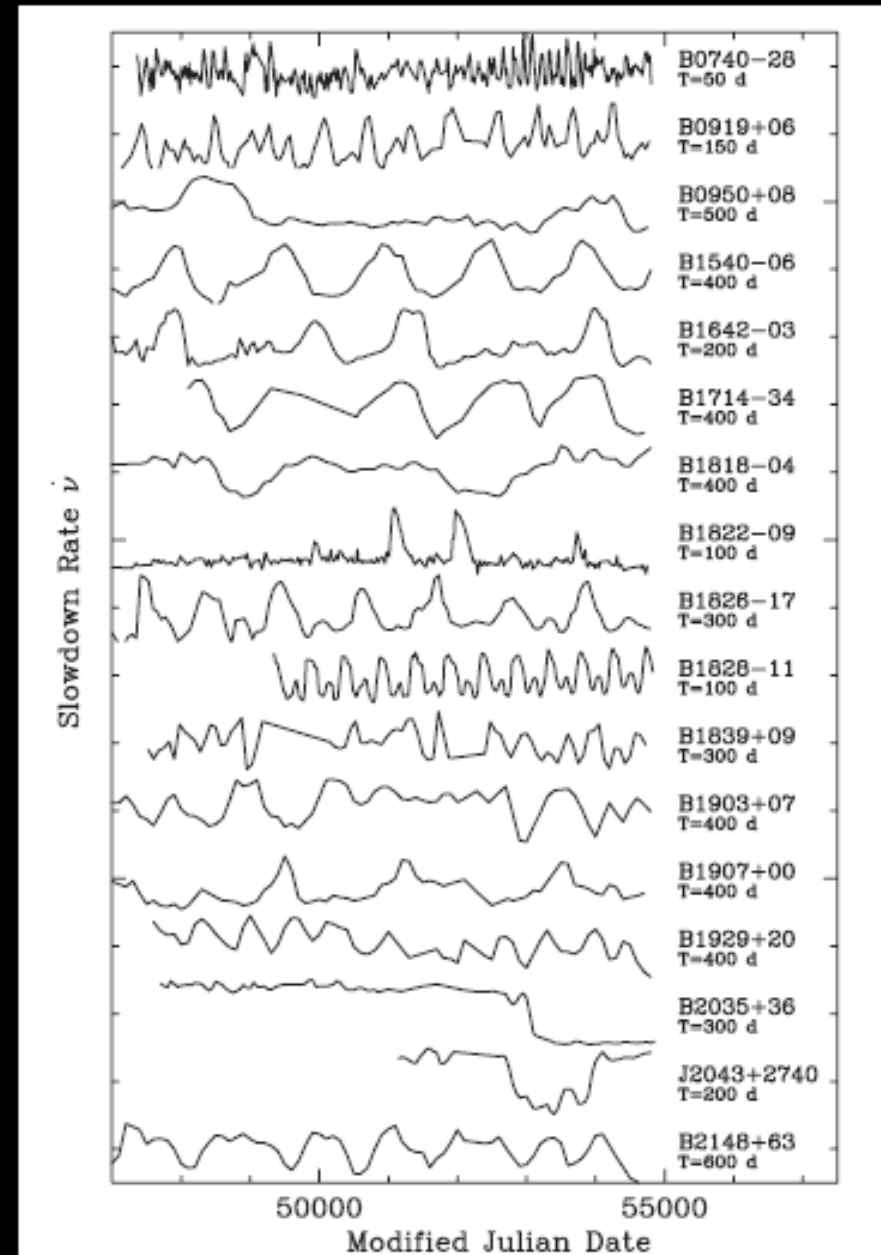
# Spin-Down $\dot{P}$

## Pulsars are 'Rotation Powered'

- A rotating magnetic dipole radiates energy
- The pulsar slows down at a rate  $\dot{P}$
- $\dot{P} = -KP^{2-n}$
- $n$  is the 'braking index' and canonically is = 3 but,
- Measured values range from 1.4 to 2.9 indicating other energy loss process are in play.

## $\dot{P}$ itself varies over time:

- Change in  $\dot{P}$  over 27 years for 17 pulsars
- ?





# Pulse Time-of-Arrival

- Individual pulses may not be identical but integrated pulse profiles are very stable between Observations
  - Profile Stabilisation Time: few 100's to 1000s of pulse periods.
- Integrated pulses profiles are not simple shapes so which point to pick as the reference point?
- **Template:**
- Create a template model that will describe the integrated pulse profile
- Cross Correlate pulse and template

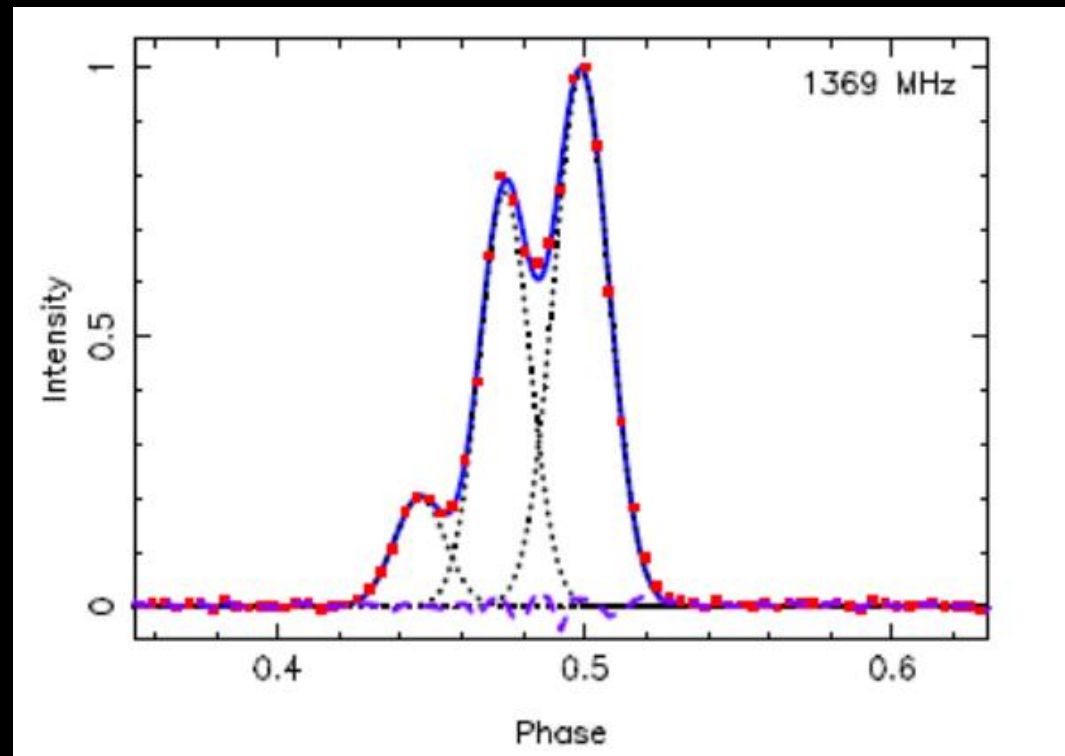


Image credit : Yuan, Manchester, Wang, et al. 2016

# Template

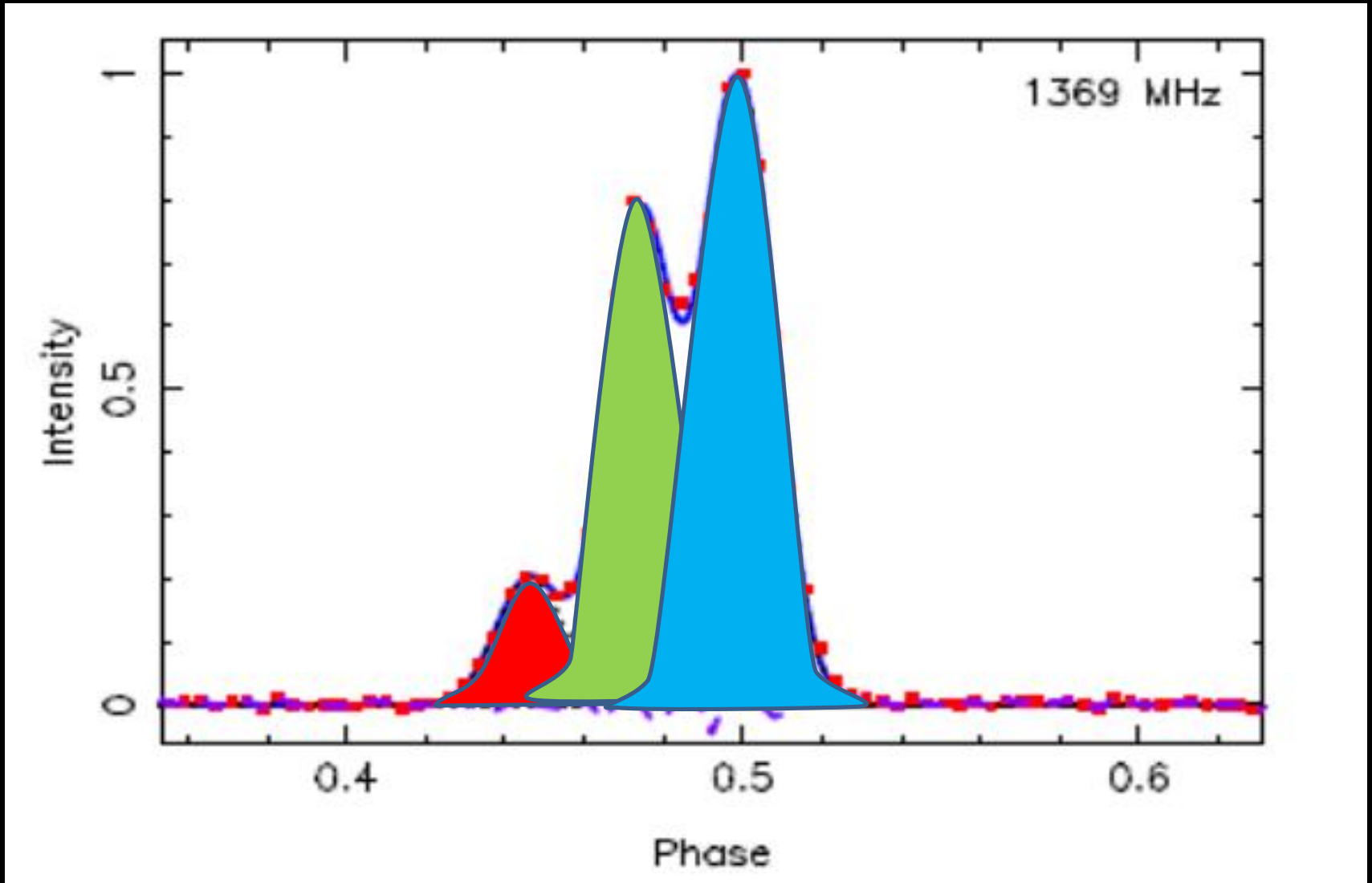
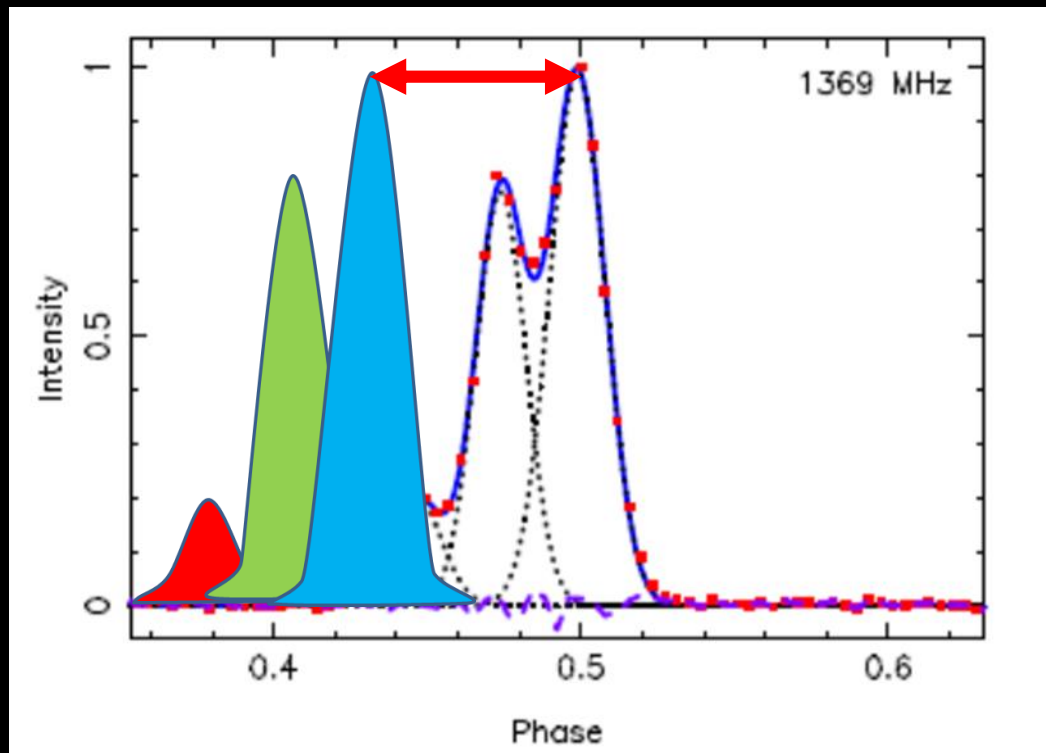


Image credit : Adapted from Yuan, Manchester, Wang, et al. 2016

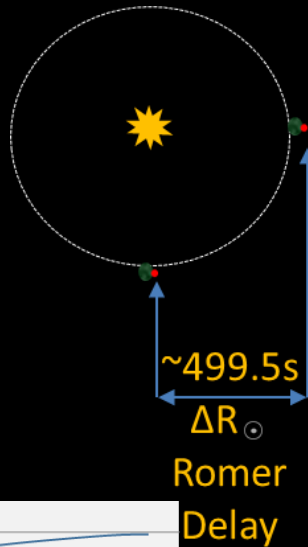
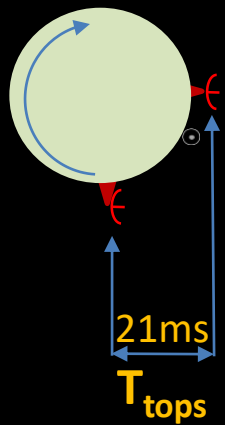
# Time of Arrival

- **Determine the ToA of an integrated pulse profile by the reference to the template model**
  - Cross Correlation gives a phase difference between the expected and measured phase
  - Convert to a time and add to time of first bin to get the ToA

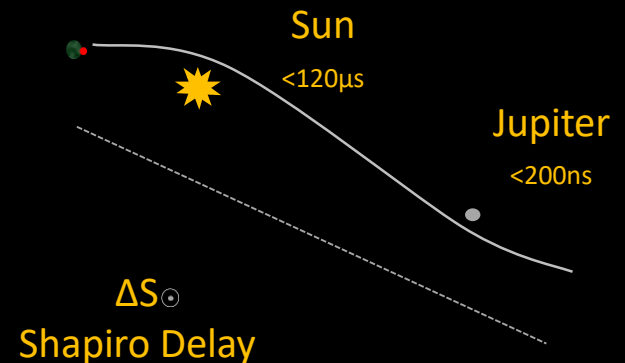
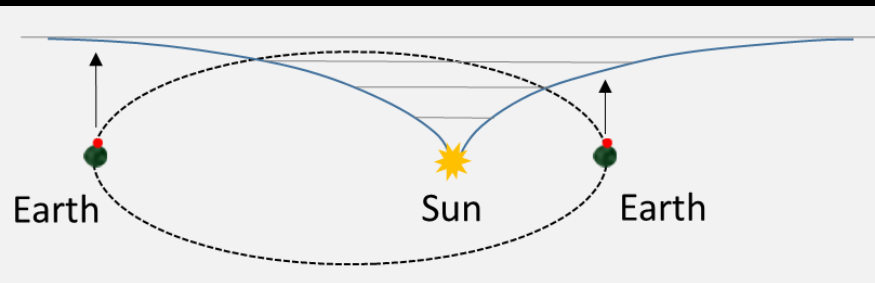


# Corrections to ToA

$$t_{SSB} = t_{topo} + t_{corr} + \Delta R_{\odot} + \Delta S_{\odot} + \Delta E_{\odot}$$



$\sim 1.6ms$   
 $\Delta E_{\odot}$  Einstein Delay



Correction	Typical value/range
Observatory clock to TT	1 $\mu s$
Hydrostatic tropospheric delay	10 ns
Zenith wet delay	1.5 ns
IAU precession/nutation	$\sim 5$ ns
Polar motion	60 ns
$\Delta UT1$	1 $\mu s$
Einstein delay	1.6 ms
Roemer delay	500 s
Shapiro delay due to Sun	112 $\mu s$
Shapiro delay due to Venus	0.5 ns
Shapiro delay due to Jupiter	180 ns
Shapiro delay due to Saturn	58 ns
Shapiro delay due to Uranus	10 ns
Shapiro delay due to Neptune	12 ns
Second-order Solar Shapiro delay	9 ns
Interplanetary medium dispersion delay	100 ns <sup>b</sup>
ISM dispersion delay	$\sim 1$ s <sup>b</sup>

# Timing Model and Residuals

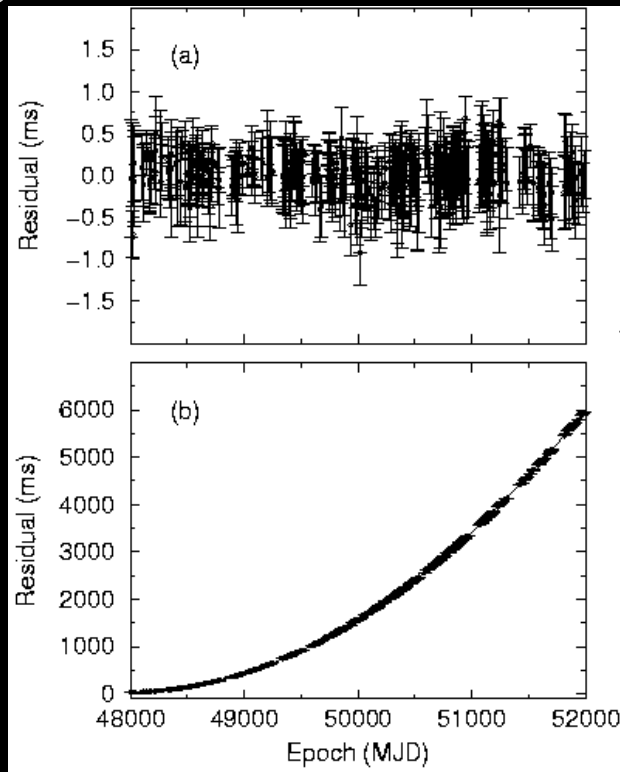
- **A timing model allows us to calculate the expected arrival of**
  - Pulse number 1,000,000 (day 11)
  - Pulse number 10,000,000 (day 115)
  - Pulse number 1,000,000,000 (day 11,575 – in year 31)
- **Iterative process starting with a few closely grouped observations**
  - Extend the time duration and precision as more data is available
- **Look out for**
  - Phase coherence
  - Geometric and relativistic effects
  - Glitches
- **The ToA is compared with that predicted by the timing model, and we can determine the error in the model.**
  - ‘Residual error’

# Timing Residuals from PSR B1133+16

$$t_{\text{SSB}} = t_{\text{topo}} + t_{\text{corr}} + \Delta_R + \Delta_S + \Delta_E$$

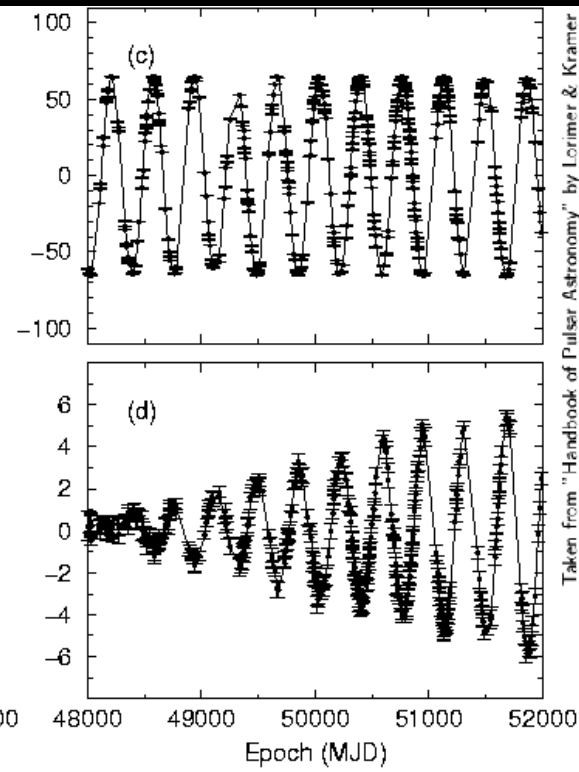
⊙ ⊙ ⊙

a) Perfect fit – random distribution =  
'Timing noise'



b) Parabolic increase -  $\dot{P}$  underestimated  
Timing model incorrect

c) Position error – sinusoidal error  
with a period of 1 year = Romer delay



d) Position not corrected for proper  
motion – Romer delay error

Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

# Phase Coherence

- Simply we have a timing solution that is accurate enough that the accumulated uncertainties do not exceed one pulse period.
- **Building a Timing Model**
  - Start with Spin period and Pulse reference phase (template)
  - Spin frequency derivative ( $\dot{P}$ )
  - Position
- Precision improves with
  - Data span (time) and
  - Frequency of observations.
- *Pulsars in Binary Systems have another set of orbital parameters to characterise*

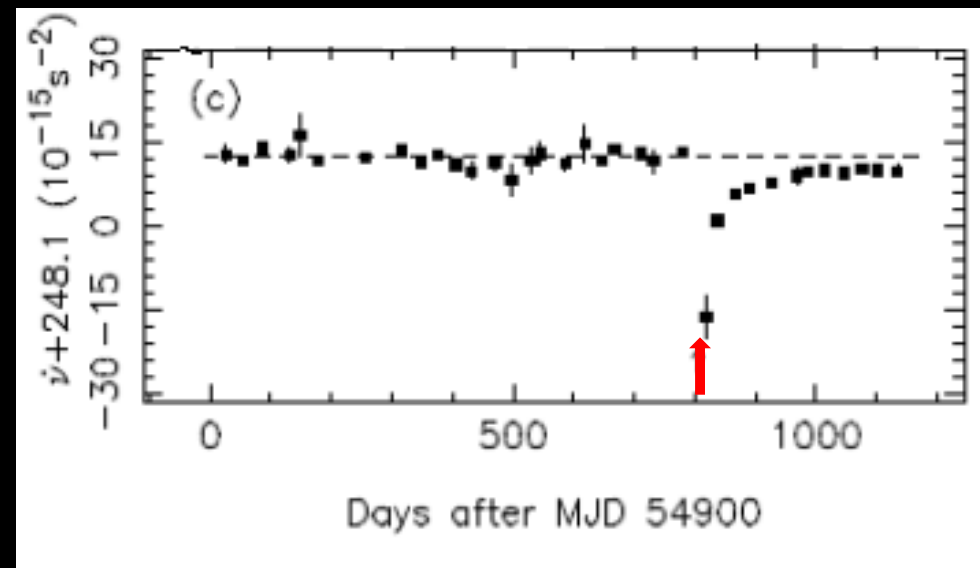
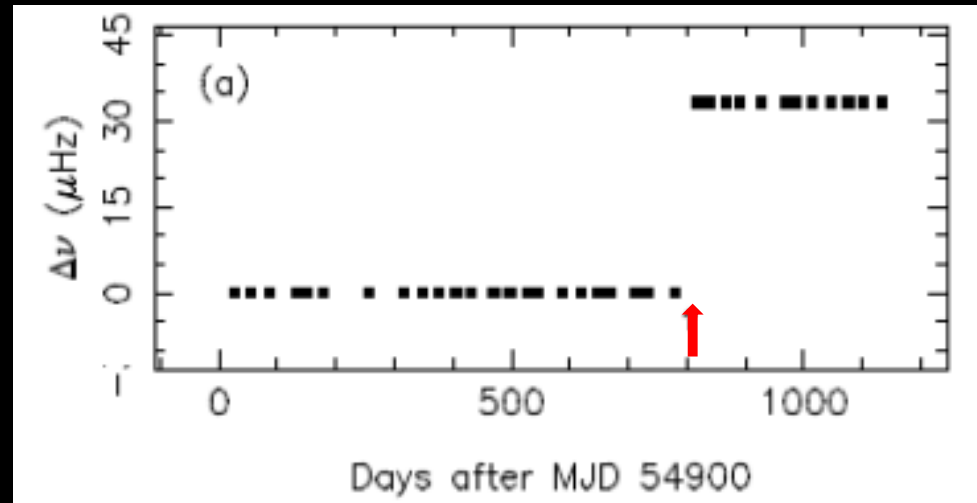


# Glitches

- **A glitch is a sudden step change in spin frequency**
  - The pulsar spins faster
  - The spin down rate ( $\dot{P}$ ) increases and then decays back to the original value.
  
- **Glitch in PSRJ1757-2421**
  - at  $\sim$  MJD 55700 (May 16 – 27 2011)
  - Fractional increase of  $\Delta\nu_g / \nu \sim 7.8 \times 10^{-6}$
  - $\nu = 4.271\,592\,656\,55(4) \text{ s}^{-1}$
  - $\Delta\nu_g = 4.271\,573\,098\,0(8) \text{ s}^{-1}$

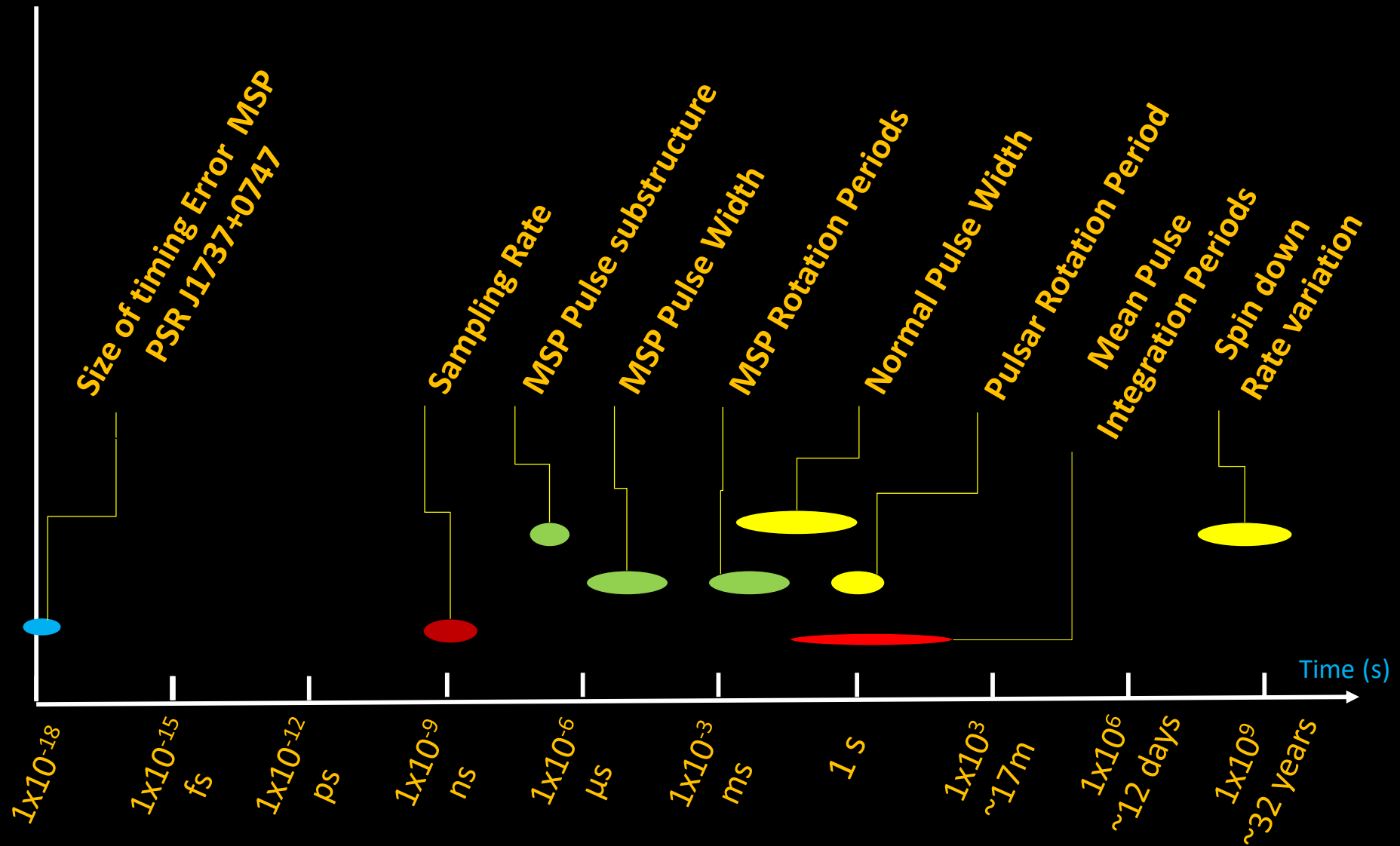
# Glitch in PSR J1757-2421

- The top panel shows the spin-frequency residuals relative to the pre-glitch Spindown solution;
- The bottom panel shows the variations of  $\dot{P}$ , and the horizontal dashes indicate the average pre-glitch level.



*The glitch epoch is indicated by a red arrow.*

# Timescales



*Thank you*

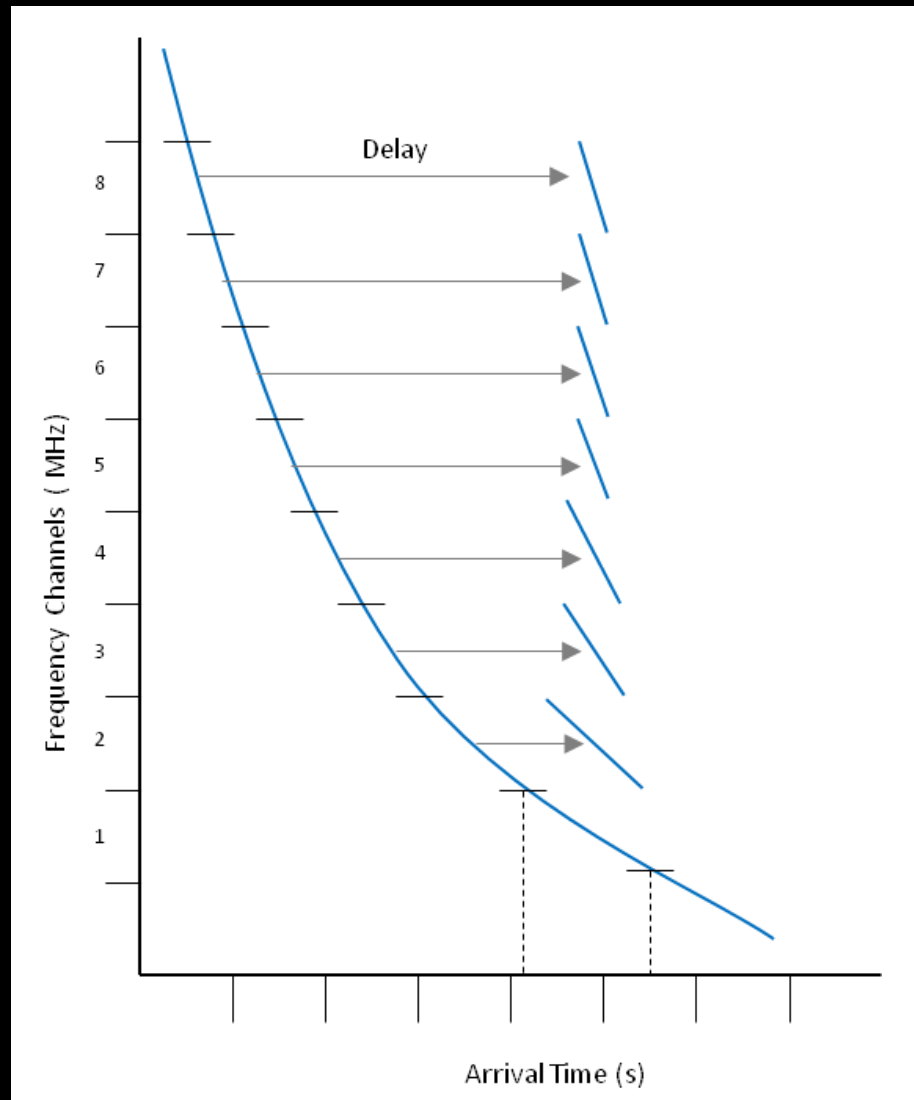
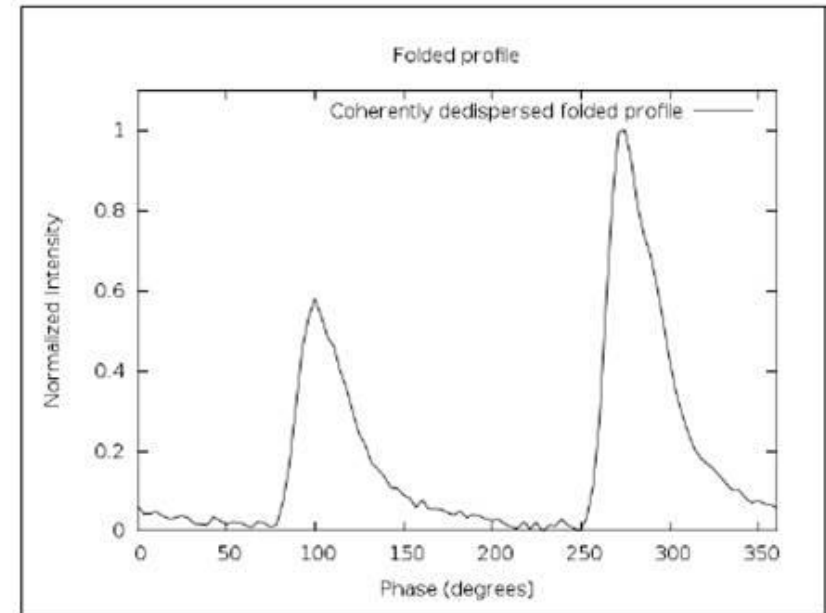
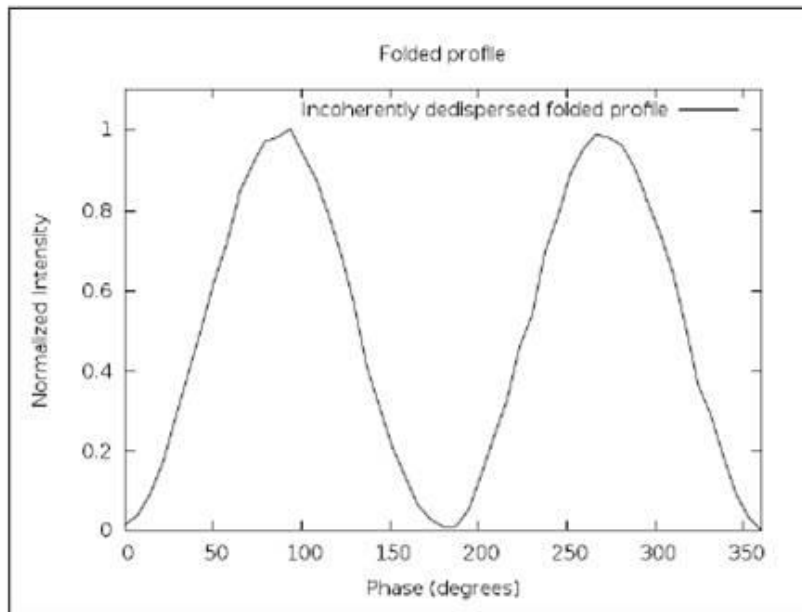


Figure 2.5.2 Incoherent De-dispersion. The signal phase in adjacent frequency channels is delayed by an amount depending on the channel centre frequency. The uncorrected frequency dispersion in each channel (indicated by the slope of the blue line segments) gives rise to pulse smearing in the pulse profile. (Adapted from Figure 3.6, Lyne & Graham-Smith (2012)).

# Figures



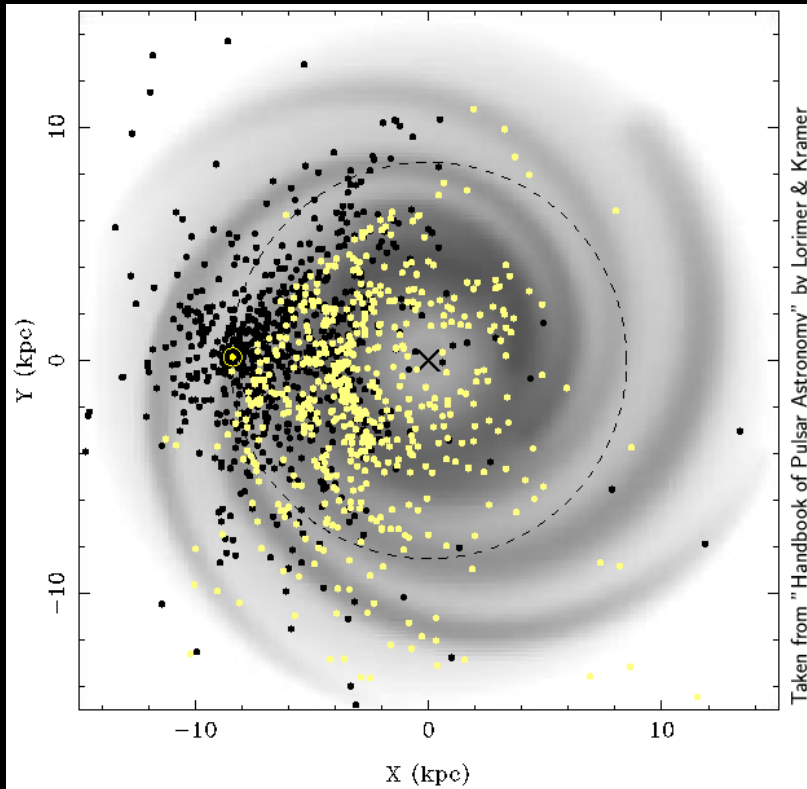
(a) Incoherently dedispersed and normalized folded profile for B1937+21. (b) Coherently dedispersed and normalized folded profile for B1937+21.

Figure 2.5.3 Comparison of incoherent and coherent de-dispersed and folded pulse profiles for PSR B1937+21 (period of 1.55ms and a DM of 71 pc cm<sup>-3</sup>). The effect of pulse smearing in the incoherently dedispersed mean pulse profile is evident in (a) when compared with the coherently dedispersed mean pulse profile in (b). Panel (b) also shows evidence of the effect of scattering in the long tails on the right hand side of the pulse profile. Data observed with the GMRT @ 325 MHz and a bandwidth of 16

(from Figure 1 De & Gupta, 2015).

# ISM Electron density distribution models

NE2001 Cordes and Lazio (2002a)



- Pulsars detected at ~430MHz are black dots, at ~1.4GHz yellow dots
- Darker grey corresponds to higher electron number density

YMW16 (2016)

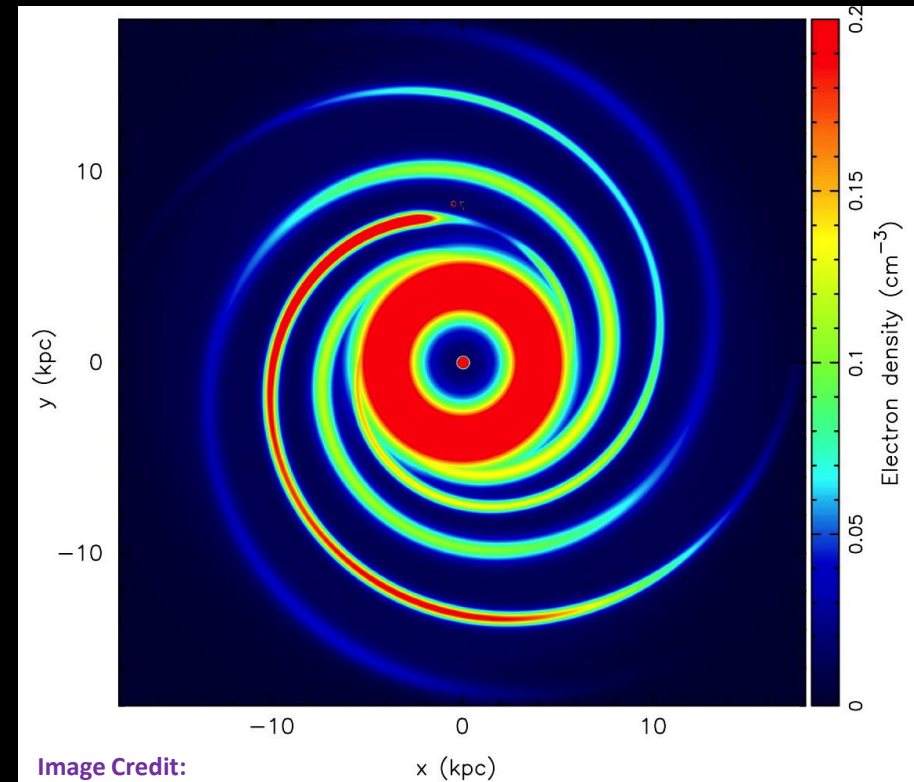


Image Credit:

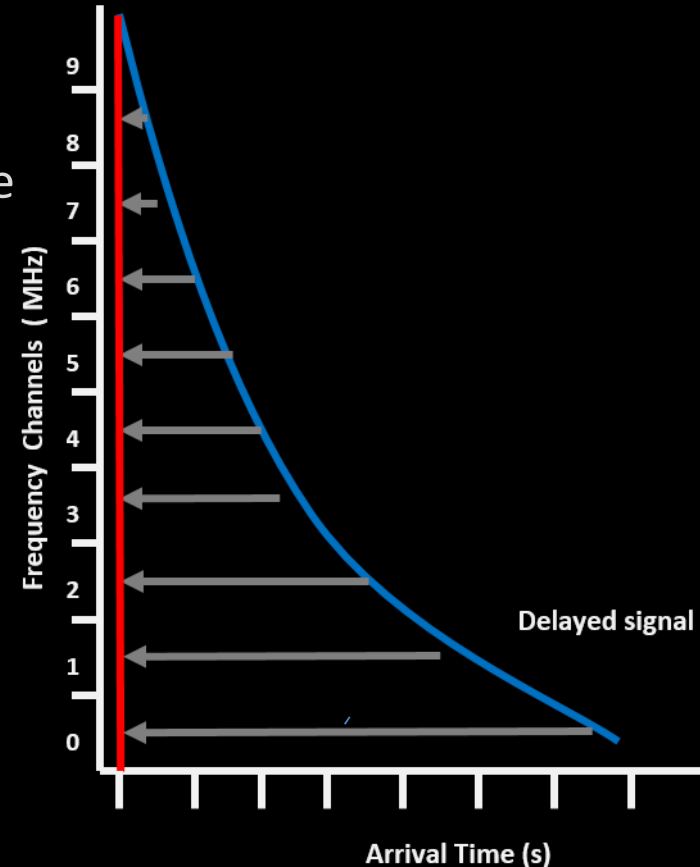
<https://www.atnf.csiro.au/research/pulsar/ymw16/>

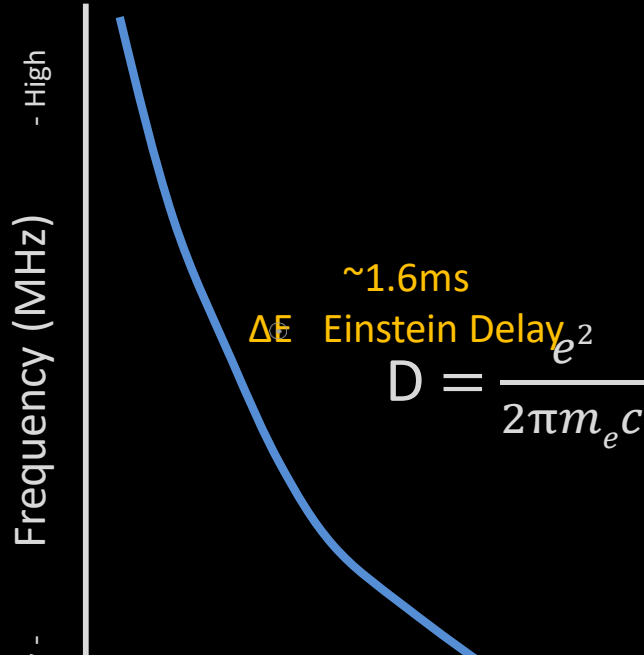
- YMW16, Yao, Manchester and Wang Astrophysical Journal, vol. 835, eid 29 (2017) (via [arXiv](https://arxiv.org/abs/1610.09448v2) (1610.09448v2)).
- YMW16 is the first electron-density model to estimate extragalactic pulsar and FRB distances.



# Coherent Dedispersion

- **Coherent Dedispersion**
  - Use a chirp filter to reverse the effect of the Interstellar Medium on the pulsar signal.
  - Gives a much better signal to noise and pulse shape resolution
  - Use a FFT to carry out dedispersion and filterbank synthesis
  - The processing load is proportional to the Fourier transform length.
- **Undertake processing in real time**
  - Alternatively store the input data and process later
  - Accumulating at 1.536 GBytes per second\*





~1.6ms  
 $\Delta E$  Einstein Delay  $e^2$   
 $D = \frac{e^2}{2\pi m_e c}$

$$t = \left( \int_0^d \frac{dl}{v_g} \right) - \frac{d}{c} \quad \text{s} \quad (a)$$

$$\Delta t = D \times (f_1^{-2} - f_2^{-2}) \times DM \quad \text{s} \quad (b)$$

$$D \approx 4.148808 \pm 0.00003 \times 10^3 \text{ MHz}^2 \text{ pc}^{-1} \text{ cm}^3 \text{ s} \quad (c)$$

$$DM = \int_0^d n_e dl \quad \text{cm}^{-3} \text{ pc} \quad (d)$$

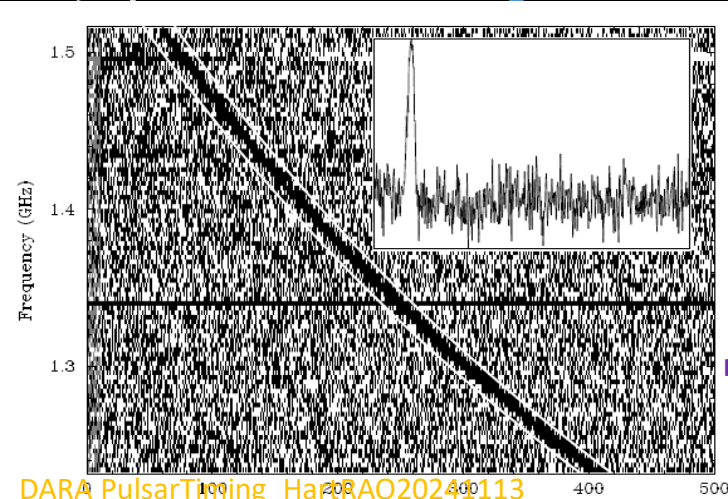


Image Credit: Handbook of  
Pulsar Astronomy  
Lorimer & Kramer

$\Delta t$  = time delay

$V_g$  = group velocity through a plasma

$D$  = Dispersion Constant

$DM$  = Dispersion Measure

$e$  = charge on the electron

$m_e$  = mass of the electron